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The additive structure of $\tilde{K}(S^{4n+3}/Q_t)$

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§1. Introduction

Let t be a positive integer and let Q_t be the group of order $4t$ given by

$$Q_t = \{x, y: x^t = y^2, xyx = y\},$$

the group generated by two elements x and y with the relations $x^t = y^2$ and $xyx = y$, that is, Q_t is the subgroup of the unit sphere S^3 in the quaternion field H generated by the two elements

$$x = \exp(\pi \boldsymbol{i}/t) \quad \text{and} \quad y = \boldsymbol{j};$$

and $Q_1 = Z_4$ and Q_t for $t = 2^{m-1}$ $(m \geqq 2)$ is the generalized quaternion group which is denoted by H_m in [4].

Then, Q_t acts on the unit sphere S^{4n+3} in the quaternion $(n+1)$-space H^{n+1} by the diagonal action, and we have the quotient manifold

$$S^{4n+3}/Q_t \quad \text{of dimension} \quad 4n+3.$$

Some partial results on the reduced K-ring $\tilde{K}(S^{4n+3}/Q_t)$ of this manifold are obtained by [4], D. Pitt [14], T. Mormann [13] and K. Kojima. In this paper, we shall determine completely the additive structure of $\tilde{K}(S^{4n+3}/Q_t)$.

Consider the complex representations a_0, a_1 and b_1 of Q_t given by

$$\begin{cases} a_0(x) = 1, \\ a_0(y) = -1, \end{cases} \quad \begin{cases} a_1(x) = -1, \\ a_1(y) = \begin{cases} i & \text{if } t \text{ is odd}, \\ 1 & \text{if } t \text{ is even}, \end{cases} \end{cases} \quad \begin{cases} b_1(x) = \begin{pmatrix} x & 0 \\ 0 & x^{-1} \end{pmatrix}, \\ b_1(y) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \end{cases}$$

and the elements

$$(1.1) \qquad \alpha_i = \xi(a_i - 1), \quad \beta_1 = \xi(b_1 - 2) \quad \text{in} \quad \tilde{K}(S^{4n+3}/Q_t) \qquad (\text{cf. } (3.3)),$$

where ξ is the natural ring homomorphism of the representation ring of Q_t to $\tilde{K}(S^{4n+3}/Q_t)$. Furthermore, consider the following subgroups of Q_t:

$$(1.2) \qquad G_0 = Q_r \text{ generated by } x^q \text{ and } y, \qquad G_1 = Z_q \text{ generated by } x^{2r},$$

where $t = rq$, $r = 2^{m-1}$, $m \geqq 1$ and q is odd. Then, we have the ring homomorphisms