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Notes on non-discrete subgroups of $\hat{U}(1, n; F)$

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1. Introduction

Let F denote the field R of real numbers, the field C of complex numbers, or the division ring of real quaternions K. Let $V=V^{1,n}(F)$ denote the (right) vector space F^{n+1}, together with the unitary structure defined by the F-Hermitian form

$$\Phi(z, w) = -\overline{z_0}w_0 + \overline{z_1}w_1 + \cdots + \overline{z_n}w_n$$

for $z=(z_0, z_1,\dots, z_n)$ and $w=(w_0, w_1,\dots, w_n)$. An automorphism g of V, that is, an F-linear bijection of V onto V such that $\Phi(g(z), g(w))=\Phi(z, w)$ for $z, w\in V$, will be called a unitary transformation. We denote the group of all unitary transformations by $U(1, n; F)$. Let $\{e_0, e_1,\dots, e_n\}$ be the standard basis in V, and set $\hat{e}_0=(e_0-e_1)(1/\sqrt{2})$, $\hat{e}_1=(e_0+e_1)(1/\sqrt{2})$ and $\hat{e}_k=e_k$ for $2\leqq k\leqq n$. Let D be the matrix which changes the basis $\{e_0, e_1,\dots, e_n\}$ into the basis $\{\hat{e}_0, \hat{e}_1,\dots, \hat{e}_n\}$. Let $\hat{U}(1, n; F)=D^{-1}U(1, n; F)D$. $\hat{U}(1, n; F)$ is the automorphism group of the Hermitian form

$$\tilde{\Phi}(z, w) = -(\overline{z_0}w_1+\overline{z_1}w_0) + \overline{z_2}w_2 + \cdots + \overline{z_n}w_n$$

for $z, w\in V$.

In the study of kleinian groups one is concerned with sufficient conditions for subgroups of Möbius transformations to be non-discrete (cf. [2]). Our purpose here is to give similar conditions for subgroups of $\hat{U}(1, n; F)$ to be non-discrete.

2. Preliminaries

Let $V_-=\{z\in V: \Phi(z, z)<0\}$ and $\hat{V}_-=D^{-1}(V_-)$. Obviously $\hat{V}_-$ is invariant under $\hat{U}(1, n; F)$. Let $P(V)$ be the projective space obtained from V, that is, the quotient space $V-\{0\}$ with respect to the equivalence relation: $u\sim v$ if there exists $\lambda\in F-\{0\}$ such that $u=v\lambda$. Let P: $V-\{0\}\to P(V)$ denote the projection map. We denote $P(\hat{V}_-)$ by Σ. Let $\bar{\Sigma}$ be the closure of Σ in the projective space. We shall view that each element of $\hat{U}(1, n; F)$ operates in $\bar{\Sigma}$. Let $G_0=\{g\in \hat{U}(1, n; F): g(P(\hat{e}_0))=P(\hat{e}_0)\}$, $G_\infty=\{g\in \hat{U}(1, n; F): g(P(\hat{e}_1))=P(\hat{e}_1)\}$ and $G_{0,\infty}=G_0\cap G_\infty$. The general form of elements in G_∞ is shown in [1; Lemma