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Extension problem of diffeomorphisms of a 3-torus over some 4-manifolds

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Introduction

Let T^3 denote a 3-torus $S^1 \times S^1 \times S^1$. Let $W(\ell, m, n)$ be the 4-manifold obtained from $T^3 \times [0, 1]$ by attaching three 2-handles along the three standard generators S^1 in $T^3 \times 1$ with framing numbers ℓ, m and n (see §1 for the precise definition). Then, $\partial W(\ell, m, n) = \partial_0 W(\ell, m, n) \cup \partial_1 W(\ell, m, n) = T^3 \times 0 \cup H(\ell, m, n)$ and $\pi_1(H(\ell, m, n)) = \langle \alpha, \beta, \gamma;\ \alpha = (\beta^{-1}\gamma\beta\gamma^{-1})^{\ell},\ \beta = (\gamma^{-1}\alpha\gamma\alpha^{-1})^{m},\ \gamma = (\alpha^{-1}\beta\alpha\beta^{-1})^{n} \rangle$. In particular, $H(\ell, m, n)$ is a homology 3-sphere. It is known that $H(0, m, n)$ is diffeomorphic to S^3 and $H(1, 1, 1)$ is the Poincaré homology 3-sphere. We refer the reader to [5], in which Y. Matsumoto proves some facts about $H(\ell, m, n)$ including the claims that the author made before.

We shall prove the following two theorems. Let $SDiff(T^3)$ denote the group of all orientation preserving diffeomorphisms of T^3. For an $f \in SDiff(T^3)$ we consider the matrix $f_* \in SL(3, \boldsymbol{Z})$ which is defined as the induced automorphism f_* of $H_1(T^3)$ with respect to the basis consisting of the classes of three standard generators.

Since T^3 is an irreducible and sufficiently large 3-manifold without boundary, $f_* = g_*$ implies that f and g are mutually isotopic by the theorem of Waldhausen [9].

THEOREM 1. *Let $f \in SDiff(T^3)$. Then, there exists an $F \in SDiff(W(1, 1, 1))$ such that $F|T^3 \times 0 = f$ and $F|H(1, 1, 1) = id$.*

THEOREM 2. *Let $f \in SDiff(T^3)$. Then, there exists an $F \in SDiff(W(0, 0, 0))$ satisfying $F|T^3 \times 0 = f$ and $F|H(0, 0, 0) = id$ if and only if f_* belongs to the subgroup $G = \{(a_{ij}) \in SL(3, \boldsymbol{Z});\ a_{1j} + a_{2j} + a_{3j} \equiv 1 \bmod 2\ (j = 1, 2, 3)\}$.*

REMARK. If we replace $W(0, 0, 0)$ with $W(0, m, n)$, we should replace G with gGg^{-1} where $g = \begin{pmatrix} 1 & m & n \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

As an application of Theorem 1, we have the following theorems. Take a non-singular algebraic curve C of degree 3 in the complex projective plane P^2. Then, C is diffeomorphic to a 2-torus T^2 and the self-intersection number $[C]^2 = 9$.