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On bounded entire solutions of semilinear elliptic equations

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1. Introduction

This paper is concerned with bounded solutions of the second order semilinear elliptic equations

$$\Delta u + \phi(x)u^{\gamma} = 0 \tag{1.1}$$

and

$$\Delta u + \phi(x)e^{u} = 0 \tag{1.2}$$

in the entire Euclidean space R^n, $n \geq 3$, where Δ denotes the n-dimensional Laplace operator, $\phi(x)$ is a locally Hölder continuous function in R^n and γ is a nonzero constant.

The problems of existence and nonexistence of entire solutions of semilinear elliptic equations of the form $\Delta u + f(x, u) = 0$ have been investigated by many authors; see, for example, [3], [4], [6], [8], [9] and [12]. We refer in particular to the recent papers by Ni [8, 9] in which explicit conditions are given which guarantee the existence of bounded entire solutions of (1.1) and (1.2).

Our main objective is to give conditions for the existence of bounded positive entire solutions of (1.1) and bounded entire solutions of (1.2) by means of the method of supersolutions and subsolutions. The principal device in this paper is the construction of spherically symmetric supersolutions and subsolutions for (1.1) and (1.2), and this enables us to prove the following theorems which extend considerably the basic existence results of Ni [8, 9].

THEOREM 1.1. *Suppose there exists a locally Hölder continuous function* $\phi^*(t)$ *on* $[0, \infty)$ *such that* $|\phi(x)| \leq \phi^*(|x|)$ *for all* $x \in R^n$ *and*

$$\int_0^{\infty} t\phi^*(t)dt < \infty. \tag{1.3}$$

Then (1.1) *with* $\gamma \neq 1$ *has infinitely many positive solutions which are bounded and bounded away from zero in* R^n. *Moreover, if either* $\phi(x) \geq 0$ *or* $\phi(x) \leq 0$ *for all* $x \in R^n$, *then equation* (1.1) *possesses infinitely many bounded positive solutions with the property that each of these solutions tends to a positive constant as* $|x| \to \infty$.