HIROSHIMA MATH. J.
14 (1984), 1–13

Asymptotic behavior of solutions to a Stefan problem with obstacles on the fixed boundary

Dedicated to Professor M. Ohtsuka on his 60th birthday

Nobuyuki KENMOCHI

(Received February 22, 1983)

Introduction

In this paper we consider the following problem: Find a curve $x=l(t)>0$ on $[0, \infty)$ and a function $u=u(x, t)$ on $\bar{\Omega}_l$, Ω_l being the set $\{(x, t); 0<x<l(t), 0<t<\infty\}$, satisfying

$$u_t - u_{xx} = 0 \quad \text{in} \quad \Omega_l, \tag{0.1}$$

$$u(x, 0) = u_0(x) \qquad \text{for} \quad 0 < x < l_o, \tag{0.2}$$

$$\begin{cases} u(0, t) \geqq g(t) & \text{for} \quad 0 < t < \infty, \\ u_x(0+, t) = 0 & \text{for} \quad u(0, t) > g(t), \\ u_x(0+, t) \leqq 0 & \text{for} \quad u(0, t) = g(t), \end{cases} \tag{0.3}$$

$$u(l(t), t) = 0 \qquad \text{for} \quad 0 < t < \infty, \text{ and} \tag{0.4}$$

$$\begin{cases} l'(t)(= (d/dt)l(t)) = -u_x(l(t)-, t) & \text{for} \quad 0 < t < \infty, \\ l(0) = l_o, \end{cases} \tag{0.5}$$

where l_o is a given positive number, u_o a given initial function and g is an obstacle function given on the fixed boundary $x=0$. This is regarded as a Stefan problem of type different from those treated so far. Recently the author (cf. [8]) employed a method which has evolved in the theory of nonlinear evolution equations involving time-dependent subdifferential operators in Hilbert spaces in order to show that our system admits global solutions to this problem. The purpose of this paper is to study the asymptotic behavior of the global solutions.

As to the usual Stefan problem which is described as a system with (0.3) replaced by the boundary conditions such as $u(0, t)=f(t)$ or $u_x(0+, t)=f(t)$, the existence and uniqueness as well as the asymptotic behavior of the solutions have been studied by many authors. See for instance [2–5, 9]. On the other hand, in case g is a non-negative constant function on $[0, \infty)$, Yotsutani [10, 11] discussed the system (0.1)–(0.5) and gave detailed results concerning the asymptotic