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On the group of self-homotopy equivalences of principal S^3-bundles over spheres

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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Introduction

For any (based) space X, the set $\mathscr{E}(X)$ of all homotopy classes of homotopy equivalences of X to itself forms a group under the composition of maps. The group $\mathscr{E}(X)$ has been studied by several authors. In particular, in case when X is a principal S^3-bundle over S^n, the group $\mathscr{E}(X)$ is already known for $X=SU(3)$, $Sp(2)$ by [10], for $X=S^3\times S^n$ by [13] and for $X=E_{k\omega}$ by J. W. Rutter [11], where $E_{k\omega}$ is the principal S^3-bundle over S^7 with characteristic class $k\omega\in\pi_6(S^3)$, ω a generator of $\pi_6(S^3)=Z_{12}$.

The purpose of this note is to study groups $\mathscr{E}(X)$ for principal S^3-bundles over spheres. Our main result is stated as follows:

THEOREM 3.1. *Let E_f be the principal S^3-bundle over S^n $(n\geqq 5)$ with characteristic class $f\in\pi_{n-1}(S^3)$. Assume that $\omega\circ S^3f\in f_*\pi_{n+2}(S^{n-1})$. Then we have the following exact sequence*:

$$0\to\pi_{n+3}(E_f)\to\mathscr{E}(E_f)\to\mathscr{E}(S^3\cup_f e^n)\to 1,$$

where $S^3\cup_f e^n$ is the mapping cone of f.

The group $\mathscr{E}(S^3\cup_f e^n)$ is given in [10, Th. 3.15] up to extension (see (2.2)), and the homotopy group $\pi_{n+3}(E_f)$ is studied for some f in §3.

Throughout this note, all spaces have base points, and all maps and homotopies preserve base points. For given spaces X and Y, we denote by $[X, Y]$ the set of (based) homotopy classes of maps of X to Y, and by the same letter a map $f: X\to Y$ and its homotopy class $f\in[X, Y]$.

§1. The homomorphism ϕ and its kernel

Throughout this note, let $f\in\pi_{n-1}(S^3)$ for $n\geqq 5$ be a given element, and let $X=E_f$ denote the principal S^3-bundle over S^n with characteristic class f and $K=S^3\cup_f e^n$ the mapping cone of f. Then by James-Whitehead [8], X has a cell structure given by