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Construction of solutions of a semilinear parabolic equation with the aid of the linear Boltzmann equation

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1. Introduction

Consider the Cauchy problem for a semilinear parabolic equation of the following form:

$$\text{(P)} \qquad u_t + \textstyle\sum_{i=1}^n A^i(u)_{x_i} = \nu \Delta u \quad (x \in R^n,\ t>0), \qquad u(x, 0) = u_0(x)$$

where Δ denotes the Laplacian; ν is any fixed positive number; and A^i, $i=1,\dots, n$, are C^1 functions of a single real variable. As is well known (see [8]) the solution u of the problem (P) with bounded measurable initial value u_0 converges, as $\nu \to 0$, to a global weak solution satisfying the entropy condition of the following hyperbolic problem:

$$\text{(H)} \qquad u_t + \textstyle\sum_{i=1}^n A^i(u)_{x_i} = 0 \quad (x \in R^n,\ t>0), \qquad u(x, 0) = u_0(x).$$

On the other hand, Kobayashi [7] has recently proposed an approximation scheme to the problem (H), using the solutions of the linear Boltzmann equation:

$$\text{(B)} \qquad f_t + \textstyle\sum_{i=1}^n \xi^i f_{x_i} = 0 \quad (x \in R^n,\ \xi=(\xi^1,\dots, \xi^n) \in R^n,\ t>0), \qquad f(x, \xi, 0) = f_0(x, \xi).$$

He used the function $v(x, t)=\int f(x, \xi, t)d\xi$ under a suitable choice of the initial function f_0 in order to construct approximate solutions of (H), and this procedure is an analogy of getting macroscopic quantities in fluid mechanics by integrating the corresponding microscopic ones with respect to the velocity argument. In this paper we modify the method in [7] so as to obtain approximate solutions of the parabolic problem (P).

The relationship between the initial values of (P) (or (H)) and (B) is given in the following way (compare with [7]). Take any function $\chi(\xi)$ with the following properties;