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Derivatives of Stokes multipliers

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§1. In the paper [2] we dealt with the two point connection problem for the general system of linear differential equations

$$t\frac{dx}{dt} = (A_0 + A_1 t + \cdots + A_q t^q)x, \tag{1.1}$$

where t is a complex variable and the coefficients A_i $(i=0, 1,\ldots, q)$ are n by n complex constant matrices. We shall briefly reconsider our theory of solving the connection problem in the case where there appear logarithmic solutions. And the purpose of this paper is to show that there holds the Frobenius theorem in a global sense, that is, concerning the Stokes multipliers of a set of logarithmic solutions, once the Stokes multipliers for the non-logarithmic solution are known, all the Stokes multipliers for its adjunct logarithmic solutions can be determined only by means of the differentiation with respect to the characteristic exponent.

Let

$$\begin{aligned} X(t) &= (x_0(t), x_1(t),\ldots, x_\gamma(t)) \\ &= \textstyle\sum_{m=0}^{\infty} (G_0(m, \rho_0), G_1(m, \rho_0),\ldots, G_\gamma(m, \rho_0))t^{m+\rho_0+J} \\ &= \textstyle\sum_{m=0}^{\infty} \mathfrak{G}(m, \rho_0)t^{m+\rho_0+J}, \end{aligned} \tag{1.2}$$

J being the $(\gamma+1)$ by $(\gamma+1)$ shifting matrix

$$J = \begin{pmatrix} 0 & 1 & & & 0 \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ 0 & & & & 0 \end{pmatrix},$$

be a matrix solution of (1.1) involving the logarithmic term t^J near the regular singularity $t=0$. Then the coefficient matrix $\mathfrak{G}(m, \rho_0)$ is given by a matrix solution of the following system of linear difference equations for $\rho=\rho_0$: Letting ρ be a parameter,

$$\begin{aligned} &(m+\rho-A_0)\mathfrak{G}(m, \rho) + \mathfrak{G}(m, \rho)J \\ &\quad = A_1\mathfrak{G}(m-1, \rho) + A_2\mathfrak{G}(m-2, \rho) + \cdots + A_q\mathfrak{G}(m-q, \rho). \end{aligned} \tag{1.3}$$