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Zeta function of Selberg's type for compact quotient of $SU(n, 1)$ $(n \geq 2)$

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§1. Introduction

Let R be a compact Riemann surface of genus greater than one. Let H be the upper half plane with the Poincaré metric. Then $R = H/\Gamma$ where Γ is a discrete torsion-free subgroup of $SL(2, \boldsymbol{R})$, acting freely on H via fractional linear transformations. In the well known paper [10], A. Selberg constructed a function Z_Γ associated with R for which the location and order of the zeros of Z_Γ gave us information about the topology of R and the spectrum of the Laplace-Beltrami operator on R. After that, in 1977, R. Gangolli showed how to attach a Selberg's type zeta function to a compact quotient of symmetric space of rank one in his paper [2].

By the way, these zeta functions can be thought of as providing information about the class one spectrum of G on $L^2(G/\Gamma)$, where G is a semisimple Lie group of real rank one. Namely, we decompose $L^2(G/\Gamma)$ into a direct sum of G-invariant irreducible subspaces and investigate those irreducible subspaces that contain a unique (up to scalar multiplication) K-invariant function. Here K is a maximal compact subgroup of G.

Let M be the centralizer in K of the split component of a minimal parabolic subgroup of G. Then the class one spectrum of G is contained in the representations induced from the trivial representation of M. D. Scott paid attention to this fact in [9]. Let ξ be an irreducible representation of M. As for $G = SL(2, \boldsymbol{C})$, he constructed a zeta function $Z_{\Gamma,\xi}$ which gave information about those principal series representations induced from ξ that appeared in the spectrum of G on $L^2(G/\Gamma)$.

In the present paper, we consider the analogues of those results when $G = SU(n, 1)$. That is, we construct the zeta functions $Z_{\Gamma,\tau}$ of Selberg's type for compact quotient of G, associated with the one dimensional representations τ of $K = U(n+1) \cap G$. The purpose of this paper is to show that these zeta functions have almost all the properties possessed by Selberg's one. Our main results are collected in Theorem 4.11.

In §2, we deal with preliminaries.

Making use of the trace formula, we will define the series $\eta_{\Gamma,\tau}$, the logarithmic derivative of our zeta function. On that occassion, we use the suitable function