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Supersoluble Lie algebras

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Introduction

A Lie algebra is called *supersoluble* if it has an ascending series of ideals whose factors are of dimension ≤ 1. Many authors, especially Barnes [5] and Barnes and Newell [6], have presented some properties of finite-dimensional supersoluble Lie algebras. A group is said to be supersoluble (or hypercyclic) if it has an ascending normal series whose factors are cyclic. Some properties of finite supersoluble groups have been presented in [12]. In [2] and [4] Baer has investigated supersoluble groups and has established the close connection with hypercentral groups. The purpose of this paper is first to show the connection between supersoluble Lie algebras and hypercentral Lie algebras, secondly to generalize some properties of hypercentral Lie algebras to those of supersoluble Lie algebras, and thirdly to characterize supersoluble Lie algebras by the weak idealizer condition. We shall also investigate locally supersoluble Lie algebras.

In Section 1 we shall give basic properties of supersoluble Lie algebras. Baer [2, Proposition 2] has shown that the derived group of a supersoluble group is hypercentral. In Section 2 we shall show the Lie analogue of this and characterize the Hirsch-Plotkin radical of a supersoluble Lie algebra as the unique maximal hypercentral ideal. In Section 3 we shall give criteria for a supersoluble Lie algebra to be hypercentral and for a locally supersoluble Lie algebra to be locally nilpotent, by using the nonexistence of non-abelian 2-dimensional subalgebras. We shall also give a criterion for a locally finite Lie algebra over an algebraically closed field to be locally nilpotent. It is known [12, p. 7] that the product of two finite supersoluble normal subgroups of a group need not be supersoluble. We shall show in Section 4 that over a field of characteristic zero the sum of two supersoluble (resp. locally supersoluble) ideals of a Lie algebra is always supersoluble (resp. locally supersoluble). We shall also investigate coalescence. It has been shown that every infinite-dimensional hypercentral (resp. locally nilpotent) Lie algebra has an infinite-dimensional abelian ideal (resp. subalgebra) [1, Theorems 10.1.1 and 10.1.3]. We shall show in Section 5 that we may replace 'hypercentral' or 'nilpotent' by 'supersoluble' in the preceding assertion. Bear [4] characterized supersoluble groups and locally supersoluble groups by the weak normalizer condition. We shall consider its Lie analogue in Sections 6 and 7. Proofs are slightly different.