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A product formula approach to first order quasilinear equations

Dedicated to Professor Isao Miyadera on his 60th birthday

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Introduction

This paper is concerned with the Cauchy problem (hereafter called (CP)) for the scalar quasilinear equation

$$(DE) \qquad u_t + \textstyle\sum_{i=1}^{d} (\phi_i(u))_{x_i} = 0 \qquad \text{for} \quad t > 0,\ x \in \boldsymbol{R}^d$$

where $\phi = (\phi_1, \phi_2, \dots, \phi_d)$ is a smooth $\boldsymbol{R}^d$-valued function on $\boldsymbol{R}$ such that $\phi(0) = 0$.

We treat this problem from the point of view of the theory of nonlinear semigroups and establish a new operator theoretic algorithm for solving the problem in conjunction with product formulae. It is well-known that solutions of (CP) can be constructed by both the method of vanishing viscosity and the finite difference method. Recently, Giga and Miyakawa proposed in [7] a new method for constructing solutions of (CP) via the iterative scheme

$$(0.1) \qquad u_{k+1} = C_h u_k, \quad k = 0, 1, 2, \dots,$$

where the operators C_h, $h > 0$, are defined by

$$(0.2) \qquad (C_h u)(x) = \int_{\boldsymbol{R}} 2^{-1}(\operatorname{sign}(u(x - h\phi'(\xi)) - \xi) + \operatorname{sign}(\xi))d\zeta$$

for $x \in \boldsymbol{R}^d$, where h stands for a mesh size of time difference.

Let $u(t, x)$ be a function of $(t, x) \in (0, \infty) \times \boldsymbol{R}^d$ and $f(t, x, \xi)$ the function of $(t, x, \xi) \in (0, \infty) \times \boldsymbol{R}^d \times \boldsymbol{R}$ defined by

$$f(t, x, \xi) = 2^{-1}(\operatorname{sign}(u(t, x) - \xi) + \operatorname{sign}(\xi)),$$

where ξ is understood to mean a parameter varying over $\boldsymbol{R}$. Then the function u and f satisfies the relation

$$u(t, x) = \int_{\boldsymbol{R}} f(t, x, \xi)d\xi$$

and

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