HIROSHIMA MATH. J.
15 (1985), 75–79

Notes on Wishartness and independence of multivariate quadratic forms in correlated normal vectors

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(Received July 11, 1984)

Summary. This note concerns with a multivariate analogue of Ogasawara and Takahashi's theorem for the distribution of a quadratic form in correlated normal variates. Also we discuss on the independence of two such multivariate quadratic forms.

1. Introduction

Quadratic forms in univariate normal variates have been discussed for their distributions and independence between them by many authors, for example, Cochran [1], Craig [2], [3], Hotelling [5], Matusita [10], Sakamoto [15], Ogawa [13], [14], Kawada [6], Lancaster [9], Laha [8], and so on. Khatri [7] and Hogg [4] have extended the discussion to the multivariate case. Ogasawara and Takahashi [12] have considered the distribution of a quadratic form in correlated univariate normal variates, where their correlation matrix is positive semi-definite (p.s.d.). Khatri [7] and Hogg [4] have also treated the correlated and multivariate case but their correlation matrix is positive definte (p.d.).

In this note, we generalize Ogasawara and Takahashi's result to the multivariate case for the Wishartness in both central and noncentral cases. We also discuss the independence between two multivariate quadratic forms in the correlated case.

Let $V \sim W_p(\Sigma, n)$, $n \geq p$. Then it is well known that $\boldsymbol{a}'V\boldsymbol{a}/\boldsymbol{a}'\Sigma\boldsymbol{a} \sim \chi^2$ for all $\boldsymbol{a}$ such that $\boldsymbol{a}'\Sigma\boldsymbol{a} \neq 0$. If the converse of this proposition holds, multivariate generalization of Ogasawara and Takahashi's theorem is immediately obtained and nothing remains to do. It is however known from the counter-example given by Mitra [11] that the converse is not true. Thus we need to give a seperate proof of this generalization.

We also note on the independence of two multivariate quadratic forms in the correlated case, where the correlation matrix is p.s.d.

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*) This research was supported in part by Grant-in Aid for Scientific Research of Education, Science and Culture under contract number 321–6059–59530012.