

## Study of three-dimensional algebras with straightening laws which are Gorenstein domains I

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### Introduction

The concept of ASL (algebra with straightening laws) is an axiomatization of the “straightening formula” appearing in invariant theory. This axiomatization, which is lucid and charming, associates commutative algebras with combinatorics through partially ordered sets (poset for short) and moreover, with topology through simplicial complexes.

Many interesting rings which appeared in classical invariant theory, such as coordinate rings of Grassmann varieties, determinantal and Pfaffian varieties turned out to be ASL, and we can obtain many informations concerning these rings by means of corresponding posets.

On the other hand, as far as the authors know, all the examples known when we started this work, are normal, rational over the base field and are rational singularities in characteristic zero, and D. Eisenbud has proposed a conjecture in [3] that every ASL domain on a wonderful poset should be normal with rational singularities.

However, in the course of classifying Gorenstein ASL domains of dimension 3, we have discovered examples of non-normal ASL domains on wonderful posets (cf. example g)). These are examples of non-normal Del Pezzo surfaces (a Del Pezzo surface is a projective surface  $X$  whose anti-canonical sheaf  $\omega_X^{-1}$  is ample) of arbitrary high degree and we are sure that the theory of ASL will be very helpful to construct interesting examples of rings or varieties with given properties.

Our final goal is to classify all the three dimensional homogeneous Gorenstein ASL domains over a field. Toward this goal, in this first part, we will determine all the posets on which there exist three dimensional homogeneous Gorenstein ASL domains. Moreover, in this process we will show that every three dimensional homogeneous ASL domain over a field is Cohen-Macaulay. Our fundamental method is quite elementary and its origin is in [5].

The main result in this paper is the following

**THEOREM.** *Let  $k$  be a field. The posets on which there exist three dimensional homogeneous Gorenstein ASL domains are among the followings:*