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Convergence of approximate solutions for Kac's model of the Boltzmann equation

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1. Introduction

Kac's model is a one dimensional model of the Boltzmann equation and is written as follows:

$$(1.1)\qquad \begin{cases} \partial_t F = -v\partial_x F + Q(F, F), \\ F(0, x, v) = F_0(x, v), \end{cases} \qquad (t, x, v) \in [0, \infty) \times \boldsymbol{R} \times \boldsymbol{R},$$

where $F = F(t, x, v)$ is a distribution function of particles with velocity v at time t and at position x and $\partial_t F = (\partial/\partial t)F$ etc. Q is a collision operator given by

$$Q(F, G) = (1/2)\int_{-\pi}^{\pi}\int_{\boldsymbol{R}} \{F(v_1')G(v') + F(v')G(v_1') - F(v_1)G(v) - F(v)G(v_1)\} I(\theta)d\theta dv_1,$$

where $v_1' = v\sin\theta + v_1\cos\theta$, $v' = v\cos\theta - v_1\sin\theta$ and $F(v_1') = F(t, x, v_1')$ etc.

Throughout this paper we assume that $I(\theta)$ is a non-negative integrable function on $[-\pi, \pi]$ and satisfies $I(\theta) = I(-\theta)$.

Note that the absolute Maxwellian state $g(v) = \exp(-v^2/2)/\sqrt{2\pi}$ is a stationary solution for (1.1). Putting $F = g + g^{1/2}f$ and substituting it into (1.1), we have the equation for f:

$$(1.2)\qquad \begin{cases} \partial_t f = -v\partial_x f + Lf + \Gamma(f, f) \equiv Bf + \Gamma(f, f), \\ f(0, x, v) = f_0(x, v), \end{cases}$$

where $Lf = 2g^{-1/2}Q(g, g^{1/2}f)$ and $\Gamma(f, f) = g^{-1/2}Q(g^{1/2}f, g^{1/2}f)$. According to [2], the eigenvalues $\{\lambda_n\}_{n=0}^{\infty}$ and the corresponding eigenvectors $\{e_n\}_{n=0}^{\infty}$ of the linearized collision operator L are given by

$$\lambda_0 = 0, \quad \lambda_n = \int_{-\pi}^{\pi} (\sin^n\theta + \cos^n\theta - 1)I(\theta)d\theta \quad n \geqq 1,$$

$$e_n = e_n(v) = \exp(-v^2/4)H_n(v)/\|\exp(-v^2/4)H_n(v)\|_{L^2(\boldsymbol{R}_v)} \quad n \geqq 0,$$

where $H_n(v)$ are the Hermite polynomials. In particular it should be noted that

$$\lambda_0 = \lambda_2 = 0, \quad \lambda_n < 0\ (n \neq 0, 2),\ \lim_{n\to\infty}\lambda_n = -\nu,$$