HIROSHIMA MATH. J.
21 (1991), 419–426

On homology of the double covering over the exterior of a surface in 4-sphere

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(Received May 19, 1990)

Introduction.

We consider a closed connected surface F embedded in a homology 4-sphere M^4 with normal bundle $N(F)$. Of course $N(F)$ always exists as a regular neighborhood of F in the smooth or PL category. The exterior X of F is defined by $X = M^4 - \operatorname{Int} N(F)$. If F is non-orientable (resp. orientable), then $H_1(X) \cong H^2(F) \cong \boldsymbol{Z}_2$ (resp. $\boldsymbol{Z}$) by the Alexander duality, and we have the double covering space X_2 over X associated with the kernel of the non-trivial homomorphism $\pi_1(X) \to \boldsymbol{Z}_2$ through the Hurewicz homomorphism $\pi_1(X) \to H_1(X)$. In this paper, we determine the finitely generated Λ_2-modules $H_*(X_2)$ and $H_*(X_2, \partial X_2)$. Here Λ_2 denotes the integral group ring of $\boldsymbol{Z}_2$ which is generated by t, and t acts on these homology groups by the induced isomorphism of the covering transformation.

THEOREM 1. *If F is non-orientable, we have the following.*

(1) $H_1(X_2) \cong H_1(X_2, \partial X_2) \cong \bigoplus_{i=1}^{n} \Lambda_2/(t+1, c_i)$, *where* c_i $(1 \le i \le n)$ *are odd integers.*

(2) $H_2(X_2) \cong H_2(X_2, \partial X_2) \cong \Lambda_2{}^{g-1} \oplus \Lambda_2/(t+1) \oplus H_1(X_2)$, *where g is the genus of F.*

(3) $H_i(X_2) = 0$ $(i \ge 3)$, $H_i(X_2, \partial X_2) = 0$ $(i = 0, 3$ *or* $i \ge 5)$, *and* $H_0(X_2) \cong H_4(X_2, \partial X_2) \cong \Lambda_2/(t-1)$.

THEOREM 1′. *If F is orientable, we have the following.*

(1′) $H_1(X_2, \partial X_2) \cong \bigoplus_{i=1}^{n} \Lambda_2/(t+1, c_i)$ *and* $H_1(X_2) \cong \Lambda_2/(t-1) \oplus H_1(X_2, \partial X_2)$, *where* c_i $(1 \le i \le n)$ *are odd integers.*

(2′) $H_2(X_2) \cong H_2(X_2, \partial X_2) \cong \Lambda_2^{2g} \oplus H_1(X_2, \partial X_2)$, *where g is the genus of F.*

(3′) $H_i(X_2) = 0$ $(i \ge 3)$, $H_i(X_2, \partial X_2) = 0$ $(i = 0$ *or* $i \ge 5)$, *and* $H_0(X_2) \cong H_3(X_2, \partial X_2) \cong H_4(X_2, \partial X_2) \cong \Lambda_2/(t-1)$.

REMARK. In the case that $\pi_1(X)$ is an abelian group, the above theorems are well known because F is stably unknotted (cf. [2]).