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An Algorithm for Constructing a Weight-Controlled Subset and Its Application to Graph Coloring Problem

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Contents

Intoduction

An *existential problem of a weight-controlled subset*, which is abbreviated by a WSP, is a combinatorial problem proposed in [16]. This is specified by a 5-tuple

$$\langle U, \boldsymbol{S}, \omega\colon U \times \boldsymbol{S} \to Z_{+0},\ a, b\colon \boldsymbol{S} \to Z_{+0} \rangle \tag{0.1}$$

of a finite set U, a collection $\boldsymbol{S}$ of subsets of U and non-negative integer valued functions ω, a, b, where the *weight function* satisfies

$$\begin{cases} \omega(u, s) > 0 & \text{if } u \in s, \\ \omega(u, s) = 0 & \text{otherwise,} \end{cases}$$

for any $u \in U$ and $s \in \boldsymbol{S}$. Then, the problem is to find a subset A of U satisfying

$$a(s) \leqq \Omega(A, s) = \sum_{u \in A} \omega(u, s) \leqq b(s) \qquad \text{for any } s \in \boldsymbol{S}. \tag{0.2}$$

This problem is a general form of various combinatonial problems, e.g., the problem of timetables [6, 10, 12, 16], graph colorings [2, 20, 22, 23], network flows [8] or Latin squares [21]. It is difficult in general to check all $\Omega(A, s)$ when U and $\boldsymbol{S}$ are large, and several researches to solve the problems are done in these papers.

The purpose of this paper is to propose a new efficient algorithm to solve a WSP. To do this, for a given WSP specified by (0.1), we consider a subset $B \subset U$, called to be *bounded by b*, satisfying