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Vanishing of Im J classes in the stunted quaternionic projective spaces

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§ 1. Introduction

Let HP^n be the quaternionic projective space, and

$$i\colon S^{4m} \longrightarrow HP_m^l = HP^l/HP^{m-1} = S^{4m} \cup e^{4m+4} \cup \cdots \cup e^{4l}$$

be the inclusion to the bottom sphere in the stunted space. Then the purpose of this paper is to investigate the induced homomorphism

$$(1.1) \qquad i_*\colon \pi_{4n-1}^s(S^{4(n-r)}) \longrightarrow \pi_{4n-1}^s(HP_{n-r}^l) \quad (n - r = m \leqq l)$$

of i between the stable homotopy groups on $(\operatorname{Im} J)_2$, where

(1.2) $(\operatorname{Im} J)_2$ is the 2-primary component of the image of the stable J-homomorphism $J\colon \pi_{4r-1}(SO) \to \pi_{4r-1}(\Omega^\infty S^\infty) = \pi_{4n-1}^s(S^{4(n-r)})$ $(r \geqq 1)$, and is the cyclic group of order $2^{3+\nu(r)}$ by Adams [1] and Quillen [7].

Here and throughout this paper $\nu(r) = \nu_2(r)$ denotes the exponent of 2 in the prime power decomposition of a positive integer r. Also, we put

$$(1.3) \qquad a(n, r) = \binom{n+r}{r}, \quad b(n, r) = \binom{n+r-1}{r-1}.$$

The main result is stated as follows:

THEOREM A. *The induced homomorphism i_* in* (1.1) *satisfies the following properties on* $(\operatorname{Im} J)_2$ *in* (1.2).

(i) *If $l < n$, then i_* is injective on* $(\operatorname{Im} J)_2$.

(ii) *Let $l = n$ and r be odd $\geqq 1$. Then $i_*((\operatorname{Im} J)_2)$ is 0 if $a(n, r)$ is odd,*
$Z/2$ if $b(r, r)$ is odd and $r + n \equiv 0 \bmod 4$,
$Z/4$ if $r > 1$, $b(n, r)$ is odd and $n \equiv r \equiv 1 \bmod 8$,
$Z/8$ if $r = 1$ and $n \equiv 1 \bmod 8$.

(iii) *Let $l = n$ and r be even $\geqq 2$, and assume that $b(n, r)$ is odd. Then* $\operatorname{Ker}(i_*) \cap (\operatorname{Im} J)_2$ *is*

$$0 \textit{ if } \nu(n) = \nu(r), \quad Z/2 \textit{ if } \nu(n) > \nu(r).$$