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On the construction of spherical hyperfunctions on $\boldsymbol{R}^{p+q}$

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Introduction

We consider $SO_0(p, q)$ (or $O(p, q)$-invariant solutions u of the differential equation $(p + v)u = 0$, where $P = \sum_{1 \le i \le p} (\partial/\partial x_i)^2 - \sum_{1 \le j \le q} (\partial/\partial y_j)^2$ and v is a complex number. There have appeared several papers dealing with the above solutions in the sense of distributions ([4], [9], [10], [14]). On the other hand, we find as a corollary of the result of A. Cerezo [2]: the dimension of the space of $O(p, q)$-invariant hyperfunctions u on $\boldsymbol{R}^{p+q}$ which are solutions of the equation $(P + v)u = 0$ is 2 and only $SO_0(p, q)$-invariant is 2 if $p > 1$ and $q = 1$, or $p = 1$ and $q > 1$, 4 if $p = 1$, respectively.

In this paper, we call such hyperfunctions "spherical hyperfunctions" and will give integral representations of "spherical hyperfunctions". In the paper [3], Ehrenpreis' principle says that any solution u of a differential equation $Pu = 0$ with constant coefficients has an integral representation by a suitable measure on the variety defined by the polynomial $\sigma_T(P)(i\xi)$, where $\sigma_T(P)$ is the total symbol of P. Thus spherical hyperfunctions may be represented through integrals with respect to $SO_0(p, q)$ (or $O(p, q)$)-invariant measures on the variety $\{(\xi, \eta) \in \boldsymbol{C}^{p+q}; \sum \xi_i^2 - \sum \eta_j^2 - v = 0\}$. But these integrals are not convergent at any point of $\boldsymbol{R}^{p+q}$. However, in his paper [11], Sato's idea enables us to justify these integrals. Thus we can construct spherical hyperfunctions explicitly. In this paper, when v is not 0, we give integral representations of spherical hyperfunctions except for $p > 1$ and $q = 1$. But when $p > 1$ and $q = 1$ we can construct spherical hyperfunctions in the same way as in the case of $p = 1$ and $q > 1$.

I would like to express hearty thanks to Professor K. Okamoto who taught me Sato's idea.

§0. Notations

Let $G = O(p, q)$ and $G_0 = SO_0(p, q)$ for $p \ge 1$ and $q \ge 1$. Then both G and G_0 are acting on $\boldsymbol{R}^{p+q}$ naturally. Let v be a non-zero arbitrary complex number and put $\mu = (1/2)\mathrm{Arg}(v)$ (Arg is the principal value) and $\lambda = |v|^{1/2} e^{i\mu}$, where $i = (-1)^{1/2}$. Then $-\pi/2 < \mu \le \pi/2$ and $v = \lambda^2$. Let $\mathfrak{g} = \mathfrak{so}_0(p, q)$ that is the Lie algebra of both G and G_0. Let $\mathscr{B}^G(\boldsymbol{R}^{p+q})$ $(\mathscr{B}^{G_0}(\boldsymbol{R}^{p+q}))$ be the space of