HIROSHIMA MATH. J.
15 (1985), 393–410

On minimally thin and rarefied sets in R^p, $p \geq 2$

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(Received October 31, 1983)
(Revised December 20, 1984)

§ 1. Introduction

Let $D = \{x \in \boldsymbol{R}^p : x_1 > 0\}$ where $x = (x_1, \ldots, x_p)$ and $p \geq 2$ and let ∂D be the euclidean boundary of D. If u is subharmonic in D and $y \in \partial D$, we define $u(y) = \limsup u(x)$, $x \to y$, $x \in D$. If $u \leq 0$ on ∂D and if $\sup u(x)/x_1 < \infty$, then it is known that

$$u(x)/x_1 \to \alpha, \quad x \to \infty, \quad x \in D \setminus E, \tag{1.1}$$

where the exceptional set E is minimally thin at infinity (cf. J. Lelong–Ferrand [8]). This result is best possible in the sense that the property of minimal thinness at infinity in D completely characterizes the exceptional set in question. If $p \geq 3$, it is also known that

$$(u(x) - \alpha x_1)/|x| \to 0, \quad x \to \infty, \quad x \in D \setminus F, \tag{1.2}$$

where the exceptional set F is rarefied at infinity in D (cf. Essén–Jackson [5b]).

In the present paper, we deduce precise descriptions of the geometrical properties of the exceptional sets E and F which will be new when $p = 2$ and which will be improvements of the results of Essén and Jackson on problems (1.1) and (1.2) when $p \geq 3$. Our Theorems 1, 2 and 3 are best possible of their kind and contain the earlier of results of this type which are due to Ahlfors and Heins [1], Hayman [6] and Azarin [2]. (For details on earlier work, we refer the reader to the introduction in [5b]).

We shall say that a set $E \subset D$ has a covering $\{t_n, r_n, R_n\}$ if there exists a sequence of balls $\{B_n\}$ with centers in D such that $E \subset \cup B_n$ where r_n is the radius of B_n, R_n is the distance from the origin to the center of B_n and t_n is the distance from the center of B_n to ∂D.

It is known that the subharmonic function u can be uniquely decomposed as

$$u(x) = \alpha x_1 - G\mu(x) - P\mu_1(x),$$

where α is defined in (1.1), $G\mu$ is the Green potential of a mass distribution μ

1) Research supported by the Swedish National Science Research Council.
2) Support from NSERC Grant #A7322 is acknowledged.