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A differential geometric approach to statistical inference on the basis of contrast functionals

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Summary

In this paper, we consider contrast functionals on the space of all probability measures equivalent to each other. Many examples of the contrast functional have been proposed and estimation methods based on them, called the minimum contrast estimation methods, have been investigated since the theory of estimation was initiated by R. A. Fisher. It is shown that a contrast functional generates a conjugate metric structure with a Riemannian metric and a conjugate pair of affine connections on the space. We show that this structure explains some properties of the minimum contrast estimator. In particular, the explicit formulas for the limiting information loss for the estimators are given in covariance structure models. Moreover we propose a generalized scoring method for seeking the minimum contrast estimates. It is shown that the convergence of the algorithm is affected by two geometric quantities which can be expressed in the conjugate metric structure.

0. Introduction

The concepts of information, entropy, energy, diversity, discrepancy, and divergence for random phenomena occupy a fundamental position in various fields of mathematical sciences, e.g., statistical mechanics, information theory, system and control theory, evolutional biology and statistics (see Boltzmann [14], Fisher [28], Shannon [54], Wiener [62], Kullback [39] and Simpson [56]). Although various terminologies for the concepts are used, the concepts have a common aspect which can be used for the comparison of random phenomena. In this sense, as a measure of the concepts we consider *a cortrast functional* defined on a pair of probability measures, which is positive except in the case of agreement between the two probability measures. R. A. Fisher [28] was the first to introduce a measure of information, called the Fisher information, into the theory of estimation. Since his work, statisticians have proposed various contrast functionals and investigated inference based on them (see, e.g. Mahalanobis [43], Battarcharya [13], Jeffreys [35], Haldane [30], Chernoff [18] Matusita [44], Reyni [53], Kagan [36], Csiszar [19] and Burbea and Rao [16]).

Closely related to the work mentioned above are geometric approaches to statistics. For example, consider a linear regression model