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Oscillation of functional differential equations with general deviating arguments

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Introduction

In this paper we consider linear and nonlinear functional differential equations with deviating arguments of the forms

$$\text{(LE)} \qquad L_n x(t) + \sigma \textstyle\sum_{h=1}^{N} q_h(t) x(g_h(t)) = 0,$$

$$\text{(NE)} \qquad L_n x(t) + \sigma \textstyle\sum_{h=1}^{N} q_h(t) f_h(x(g_h(t))) = 0,$$

where L_n is a disconjugate differential operator defined recursively by

$$\text{(1)} \qquad L_0 x = x, \quad L_i x = \frac{1}{p_i}\frac{d}{dt} L_{i-1} x \ (1 \le i \le n), \quad p_n \equiv 1.$$

The following conditions are assumed to hold throughout this paper:

(a) $n \ge 2$, $\sigma = \pm 1$;
(b) $p_i \in C(R_+, R_+ \backslash \{0\})$, $\int_0^\infty p_i(t) dt = \infty$ $(1 \le i \le n-1)$, $R_+ = [0, \infty)$;
(c) $q_h \in C(R_+, R_+)$, $g_h \in C(R_+, R)$, $\lim_{t \to \infty} g_h(t) = \infty$ $(1 \le h \le N)$;
(d) $f_h \in C(R, R)$ is nondecreasing and $x f_h(x) > 0$ for $x \ne 0$ $(1 \le h \le N)$.

The domain of L_n, $\mathscr{D}(L_n)$, is defined to be the set of all functions x which have the continuous "quasi-derivatives" $L_i x$, $0 \le i \le n$, on $[T_x, \infty)$. Our attention is restricted to those solutions $x \in \mathscr{D}(L_n)$ of (LE) or (NE) which satisfy

$$\sup \{|x(t)|: t \ge T\} > 0 \qquad \text{for any} \quad T \ge T_x.$$

Such a solution is said to be a proper solution. We make the standing hypothesis that (LE) or (NE) possesses proper solutions. A proper solution of (LE) or (NE) is called oscillatory if it has arbitrarily large zeros; otherwise it is called nonoscillatory.

We denote the sets of all proper solutions, all oscillatory solutions and all nonoscillatory solutions of (LE) or (NE) by $\mathscr{S}$, $\mathscr{O}$ and $\mathscr{N}$, respectively. It is clear that $\mathscr{S} = \mathscr{O} \cup \mathscr{N}$. Because of the conditions (a)–(d) $\mathscr{N}$ has a decomposition such that (see [2], [13] or [45]):

$$\mathscr{N} = \mathscr{N}_1 \cup \mathscr{N}_3 \cup \cdots \cup \mathscr{N}_{n-1} \qquad \text{if} \quad \sigma = 1 \text{ and } n \text{ is even},$$