HIROSHIMA MATH. J.
16 (1986), 189–207

A 3-component system of competition and diffusion

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(Received May 20, 1985)

Abstract

This report studies the existence of non-constant solutions of certain two-point boundary value problems for 3-component systems with a small parameter ε, under homogeneous Neumann conditions at the boundaries. This problem is related to the analysis of segregation patterns in population models of 3-competing and spatially dispersing species. It is shown that the reduced problem ($\varepsilon=0$) has many non-constant solutions exhibiting spatial segregation. Only a few of these, however, can serve as valid lowest-order approximations to solutions of the original problem when ε is non-zero but small. A singular perturbation construction clarifies which are in this category. The results of numerical computations of solutions are also illustrated.

1. Introduction

We consider populations of N species $S_1, S_2, \ldots, S_N$ in a bounded habitat, and assume that the distribution of the populations are determined by competition of Lotka-Volterra-Gause type and simple diffusion. Suppose $u_i(t, x)$ $(i=1, 2, \ldots, N)$ is the population density of the species S_i $(i=1, 2, \ldots, N)$. The spatial domain is taken to be the one-dimensional interval (0, 1). Then we have the following reaction-diffusion equations governing the evolution of the u_i:

$$\frac{\partial u_i}{\partial t} = d_i \frac{\partial^2 u_i}{\partial x^2} + (r_i - \sum_{j=1}^{N} a_{ij} u_j) u_i, \; t > 0, \; x \in (0, 1), \quad (i=1, 2, \ldots, N) \tag{1}$$

where d_i, r_i and a_{ij} $(i, j=1, 2, \ldots, N)$ are non-negative constants. In ecological terms, r_i is the intrinsic growth rate of S_i, a_{ii} is a measure of intraspecific competition of S_i, and a_{ij} $(i \neq j)$ is a measure of interspecific competition between the species. The boundary and initial conditions are taken to be

$$\frac{\partial u_i}{\partial x}(t, x) = 0, \quad t > 0, \; x = 0, 1 \tag{2}$$

and

† This work was done while the authors were at the Mathematics Research Center, Madison, Wisconsin (June 1982).