HIROSHIMA MATH. J.
**16** (1986), 161–168

# A central limit theorem of mixed type for a class of 1-dimensional transformations

Hiroshi ISHITANI

(Received May 18, 1985)

**Contents**



## §0. Introduction

Let $T$ be a nonsingular transformation on the interval [0, 1]. Many authors (cf. [2], [4], [5], [7], [12], [13], [14], [22], [25]) have investigated the following problem: under what conditions on $T$ and $f$ does the sequence of random variables $\{f(T^k x): k=0, 1,\dots\}$ satisfy a central limit theorem (c. l. t.)? Recently, J. Rousseau-Egele ([22]) obtained a c. l. t. for a class of transformations $T$ and its rate of convergence, by estimating the asymptotic behavior of the characteristic function with the help of the Perron-Frobenius operator corresponding to $T$.

Generalizing his method, we can get central limit theorems of mixed type for a certain class of transformations, which are stated in §1. That is, under suitable assumptions on $T$, $f$ and $\nu$, the distribution function $\nu\{\sum_{k=0}^{n-1} f(T^k x)/n^{1/2}<z\}$ is asymptotically a mixed normal distribution function. Central limit theorems for $\beta$-transformations, $\alpha$-continued fraction transformations, Wilkinson's piecewise linear transformations and unimodal linear transformations are given as corollaries to our theorems.

In §1 we intrdouce the notations and the assumption (A) under which our results are obtained. Then we state our theorems. We should remark that the rate of convergence given in (1.9), (1.12) and (1.14) is $O(1/n^{1/2})$ and it is best possible for c. l. t.

In §2 it is shown that the transformations, treated in the articles [2], [4], [5], [7], [12], [13], [14], [22] and [25], satisfy the condition (A). Therefore, the results in the above articles are given as corollaries to our Theorems 3 and 4. Moreover the unimodal linear transformations are discussed as the concrete