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The Fock representations of the Virasoro algebra and the Hirota equations of the modified KP hierarchies

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0. Introduction

The Virasoro algebra $\mathfrak{l}$ is the Lie algebra over the complex number field $\boldsymbol{C}$ of the form $\mathfrak{l}=\sum_{k\in Z} \boldsymbol{C}l_k \oplus \boldsymbol{C}c$
with the bracket relations

$$[l_k, l_m] = (k-m)l_{k+m} + \frac{1}{12}(k^3-k)\delta_{k+m,0}c, \quad [c, \mathfrak{l}] = \{0\}.$$

This algebra is the one-dimensional central extension of the so-called Witt algebra. The Virasoro algebra was introduced by physicists in their string theory of the elementary particles (cf. [6]). Mathematicians started to develop a representation theory of this algebra very recently.

Let us make a survey of the contents of this paper.

We recall the definition and some properties of the Virasoro algebra and the highest weight modules over it in section 1. An $\mathfrak{l}$-module M is called a "highest weight module" if there exists a non-zero vector v_o (the highest weight vector) such that 1) $U(\mathfrak{l})v_o=M$, where $U(\mathfrak{l})$ is the universal enveloping algebra of $\mathfrak{l}$, 2) there exists $\lambda\in(\boldsymbol{C}l_0\oplus\boldsymbol{C}c)^*$ (the highest weight) such that $Hv_o=\lambda(H)v_o$ for all $H\in(\boldsymbol{C}l_0\oplus\boldsymbol{C}c)$, 3) $l_kv_o=0$ for all positive k. The study of such modules was started by V. Kac ([3, 4]). He obtained the determinant formula for the matrix of the vacuum expectation values, and gave the "formal character" of some irreducible highest weight $\mathfrak{l}$-modules (THEOREM 1.1).

In section 2 we treat another kind of representations of $\mathfrak{l}$, which is called the "Fock representations" (cf. [6], [12]). Let a_j ($j\in \boldsymbol{Z}$) be the operators, acting on some "Fock space", with the following commutation relations:

(0.1) $$[a_j, a_i] = j\delta_{i+j,0}.$$

Define the operators

(0.2) $$L_k = \frac{1}{2}\sum_{j\in Z} : a_{-j}a_{j+k}:$$

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