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Positive entire solutions of superlinear elliptic equations

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1. Introduction

In this paper we consider the semilinear elliptic equation

$$\Delta u + p(|x|)u^{\gamma} = 0, \qquad x \in R^n, \tag{1}$$

where $n \geqq 3$, Δ is the n-dimensional Laplace operator, and $|x|$ denotes the Euclidean length of $x \in R^n$. It is assumed throughout that

(a) $\gamma > 1$ (namely, (1) is superlinear);

(b) p is continuous on $[0, \infty)$, differentiable on $(0, \infty)$ and $p(t) > 0$ for $t > 0$.

Our main concern is to study the existence and nonexistence of entire solutions of (1) which are radially symmetric and positive in R^n. Here, by an entire solution of (1) we mean a function $u \in C^2(R^n)$ which satisfies (1) at every point of R^n, and the radial symmetry of a function means that it depends only on $|x|$.

The principal results of this paper are as follows:

THEOREM 1 (*Existence*). *Suppose that*

$$\frac{d}{dt}\left(t^{[n+2-\gamma(n-2)]/2}p(t)\right) \leqq 0 \qquad \textit{for} \quad t > 0. \tag{2}$$

Then, for any $\alpha > 0$, *equation* (1) *has a radially symmetric positive entire solution* u *such that* $u(0) = \alpha$.

THEOREM 2 (*Nonexistence*). *Suppose that*

$$\frac{d}{dt}\left(t^{[n+2-\gamma(n-2)]/2}p(t)\right) \geqq 0 \qquad \textit{for} \quad t > 0 \tag{3}$$

and

$$t^{[n+2-\gamma(n-2)]/2}p(t) \longrightarrow \infty \quad \textit{as} \quad t \longrightarrow \infty. \tag{4}$$

Then, equation (1) *has no radially symmetric positive entire solutions.*

Since our attention is restricted to radially symmetric solutions, the problem for (1) under consideration reduces to the one-dimensional initial value problem

$$(t^{n-1}y')' + t^{n-1}p(t)y^{\gamma} = 0, \qquad t > 0, \tag{5}$$