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Generalized J-integral and three-dimensional fracture mechanics II —Surface crack problems—

Dedicated to Professor Sigeru Mizohata on his sixtieth birthday

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Introduction

The Griffith energy balace theory in fracture mechanics has been reformulated in terms of J-integrals, so that the fundamental relationship:

the energy release rate is expressed by a J-integral

holds for crack grwoth problems. As stated in the previous paper [12], this relationship is valid only for simple (two-dimensional) models of fracture. Thus, in [12], we proposed generalized J-integrals (abbreviated to GJ-integrals) and established the relationship

(*) *the energy release rate is expressed by a GJ-integral*

for a three-dimensional fracture problem in which the crack is strictly contained in the material and the stress is free on the crack surface. Here we note that the arguments in [12] are also applicable to the two-dimensional problems (see [11]).

The GJ-integral introduced in [12] is expressed as the sum of a surface integral P and a volume integral R. The term P corresponds to the original J-integral. An integral corresponding to the term R was also considered by Destuynder and Djaoua [1] for a two-dimensional problem, in which it was shown that the energy release rate can be as well expressed only by this integral. Moreover, in more general theory (cf. [13], [14]), the term R appears to play a leading part.

Another feature of J-integral is that it represents the singularity of the elastic field (see [1], [11]). For the problem considered in [12], it was shown that the singularity appears only on the edge of the crack and the GJ-integral vanishes if the singularity does not appear.

In this paper we treat the case where the crack intersects the surface of the material, and moreover the crack itself is pressurized. In practical problems, such crack arises in various cases such as pressure vessels containing surface cracks (see e.g. Kikuchi, Miyamoto and Sakaguchi [6]), and there is much practical interest in estimating safety of such cracked structure. For two-dimensional