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On the regularity of solutions of a degenerate parabolic Bellman equation

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1. Introduction

The control of diffusion processes leads to the parabolic Bellman equation of the type:

$$\sup\{u_t + Lu^\alpha - f^\alpha; \alpha \in \Lambda\} = 0 \quad \text{in} \quad Q_T, \qquad u = 0 \quad \text{on} \quad \partial_p Q_T, \tag{1.1}$$

where $Q_T = \Omega \times (0, T)$ for a smooth bounded domain $\Omega \subset \boldsymbol{R}^n$ and $T \in (0, \infty)$, $\partial_p Q_T$ denotes the parabolic boundary of Q_T, Λ denotes a set of indices, and each L^α is a second order elliptic operator of the form:

$$L^\alpha u = -a^\alpha_{ij}(x, t)\partial^2 u/\partial x_i \partial x_j + b^\alpha_i(x, t)\partial u/\partial x_i + c^\alpha(x, t)u.$$

Here and in the sequel we use the summation convention.

In case that L^α are uniformly elliptic operators and Λ is a finite set, L. C. Evans and S. Lenhart [2] have shown that there exists a unique function $u \in W^{2,1}_\infty(Q_T) \cap C^{\lambda,\lambda/2}(Q_T)$, for some $\lambda > 0$, solving (1.1).

In this paper we investigate the following problem

$$u_t + \max\{Lu - f, du - g\} = 0 \quad \text{a.e.} \quad \text{in } Q_T, \qquad u = 0 \quad \text{on} \quad \partial_p Q_T, \tag{1.2}$$

where f, g and d are given functions, and L is a second order uniformly elliptic operator. We may regard (1.2) as a special degenerate case of (1.1), that is, a couple of a nondegenerate operator, L, and a special degenerate one, d.

The plan of this paper is as follows:
Section 2 is devoted to state and prove our main results. The proofs are done via elliptic regularization and penalization (see (2.10) below). The necessary a priori estimates of solutions to the corresponding approximate problems are obtained in Section 3. In Appendix we deal with the existence and regularity of the approximate problems.

The time independent case of (1.1) has been studied by N. V. Krylov [6] and P. L. Lions [7]. The time independent equation of (1.2) is called the obstacle