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Oscillation theorems for nonlinear differential systems with general deviating arguments

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1. Introduction

The oscillation theory of nonlinear differential systems with deviating argements has been developed by many authors. Most of them have studied two-dimensional differential systems; see, for example, Kitamura and Kusano [2–4], Shevelo, Varech and Gritsai [8], and Varech and Shevelo [9, 10]. The oscillation results for n-dimensional systems with deviating arguments have been given by Foltynska and Werbowski [1], the present author [5, 6] and Šeda [7].

The purpose of this paper is to obtain oscillation criteria for the nonlinear differential system with general deviating arguments of the form:

$$(S_r)\qquad \begin{aligned} y_i'(t) &= p_i(t)f_i(y_{i+1}(h_{i+1}(t))), \qquad i = 1, 2,\dots, n-1,\\ y_n'(t) &= (-1)^r p_n(t)f_n(y_1(h_1(t))), \qquad r = 1, 2, \end{aligned}$$

where the following conditions are assumed to hold:

(1) a) p_i: $[0, \infty)\to[0, \infty)$, $i=1, 2,\dots, n$, are continuous and not identically zero on any infinite subinterval of $[0, \infty)$, and

$$\int^{\infty} p_i(t)dt = \infty, \quad i = 1, 2,\dots, n-1;$$

b) h_i: $[0, \infty)\to R$ are continuous and $\lim_{t\to\infty} h_i(t)=\infty$, $i=1,\dots, n$;

c) f_i: $R\to R$ are continuous and $uf_i(u)>0$ for $u\neq 0$, $i=1, 2,\dots, n$.

Denote by W the set of all solutions $y(t)=(y_1(t),\dots, y_n(t))$ of the system (S_r) which exist on some ray$[T_y, \infty)\subset[0, \infty)$ and satisfy $\sup\{\sum_{i=1}^n |y_i(t)|; t\geqq T\}>0$ for all $T\geqq T_y$.

DEFINITION 1. A solution $y\in W$ is called oscillatory if each component has arbitrarily large zeros.

A solution $y\in W$ is called nonoscillatory (resp. weakly nonoscillatory) if each component (resp. at least one component) is eventually of constant sign.

DEFINITION 2. We shall say that the system (S_1) has the property A if for n even every solution $y\in W$ is oscillatory and for n odd it is either oscillatory or