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Asymptotic expansion for the distribution of the discriminant function in the first order autoregressive processes

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1. Introduction

This paper is concerned with the problem of classifying a series of T observations coming from one of two first order autoregressive Gaussian processes. Let Π_j $(j=1, 2)$ denote the first order autoregressive Gaussian process which satisfies the stochastic equation

$$y_t = \alpha_j y_{t-1} + u_t \qquad (t = \cdots -1, 0, 1, \dots) \tag{1.1}$$

where $\alpha_j (|\alpha_j| < 1)$ is known, and u_t's are independent identically distributed as $N(0, \sigma_j^2)$ with known variance σ_j^2. Suppose that $\boldsymbol{y} = (y_1, \dots, y_T)'$ is a series of T observations coming from Π_1 or Π_2. It is natural to consider a classification method based on the density of $\boldsymbol{y}$. The density of $\boldsymbol{y}$ is given by

$$f(\boldsymbol{y}; \alpha_j, \sigma_j^2) = (2\pi\sigma_j^2)^{-T/2}(1-\alpha_j^2)^{1/2} \exp\left(-\frac{1}{2}\boldsymbol{y}' \Sigma_j^{-1} \boldsymbol{y}\right) \tag{1.2}$$

when $\boldsymbol{y}$ comes from Π_j, where

$$\Sigma_j^{-1} = \frac{1}{\sigma_j^2}\begin{pmatrix} 1 & -\alpha_j & & & \\ -\alpha_j & 1+\alpha_j^2 & & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & & 1+\alpha_j^2 & -\alpha_j \\ & & & -\alpha_j & 1 \end{pmatrix}.$$

Since all the parameters are known, an optimum classification rule is based on the statistic (cf. Anderson [1])

$$V = \log\frac{f(\boldsymbol{y}; \alpha_1, \sigma_1^2)}{f(\boldsymbol{y}; \alpha_2, \sigma_2^2)} = Z - \frac{1}{2}\left\{T \log\frac{\sigma_1^2}{\sigma_2^2} - \log\frac{1-\alpha_1^2}{1-\alpha_2^2}\right\}, \tag{1.3}$$

where

$$Z = \frac{1}{2}\boldsymbol{y}'(\Sigma_2^{-1} - \Sigma_1^{-1})\boldsymbol{y}. \tag{1.4}$$