HIROSHIMA MATH. J.
16 (1986), 617–624

Nonoscillation of nonlinear first order differential equations with forcing term

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(Received October 1, 1985)
(Revised January 18, 1986)

1. Introduction

We consider the following first order differential equations

$$x'(t) + p(t)f(x(t)) = q(t), \quad t \geqq a, \tag{1}$$

and

$$x'(t) + p(t)f(x(t)) = 0, \quad t \geqq a, \tag{2}$$

where $p \in C[[a, \infty), R]$, $q \in C[[a, \infty), R]$ and $f \in C[R, R]$, $R=(-\infty, \infty)$. A solution $x(t)$ of (1) is called *oscillatory* if $x(t)$ has zeros for arbitrarily large t; otherwise, a solution $x(t)$ is said to be *nonoscillatory*. Equation (1) is nonoscillatory if every solution of (1) is nonoscillatory. The oscillatory properties of the first order functional differential equation

$$x'(t) + p(t)f(x(t-\tau)) = q(t) \tag{3}$$

are investigated by some authors (Cf. [3] and [4]). But there is scarce litrature on the ordinary case of (1) (Cf. [5]). In this paper, we mainly propose a theorem for nonoscillation of (1).

2. The unforced case

THEOREM 1. *Suppose that $f(x)=0$ for $x=0$, $f(x)\neq 0$ for $x\neq 0$ and $|p(t)|>0$ on $[a, \infty)$. Then every solution of* (2) *has at most one zero.*

PROOF. Assume that $x(t)$ is a solution of (2) which has two consecutive zeros t_1, t_2 with the property

$$x(t_1) = x(t_2) = 0 \qquad \text{for} \quad a \leqq t_1 < t_2.$$

Let $|x(t)|>0$ for $t_1<t<t_2$. By Rolle's theorem, we can take a τ such that $x'(\tau)=0$, $t_1<\tau<t_2$. From (2) we obtain

$$0 = x'(\tau) = -\, p(\tau)f(x(\tau)).$$