HIROSHIMA MATH. J.
16 (1986), 597–606

Global existence of solutions of mixed sublinear-superlinear differential equations

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(Received November 16, 1985)

1. Introduction

Let L_n be the general disconjugate operator

$$L_n = \frac{1}{p_n(t)}\frac{d}{dt}\frac{1}{p_{n-1}(t)}\frac{d}{dt}\cdots\frac{d}{dt}\frac{1}{p_1(t)}\frac{d}{dt}\frac{\cdot}{p_0(t)} \quad (n \geqq 2), \tag{1}$$

where p_i: $[0, \infty) \to (0, \infty)$, $0 \leqq i \leqq n$, are continuous. Let (1) be in canonical form [10] at ∞; i.e.,

$$\int_0^\infty p_i(t)dt = \infty, \qquad 1 \leqq i \leqq n-1. \tag{2}$$

Consider the mixed sublinear-superlinear differential equation

$$L_n y + [a(t)|y|^\alpha + b(t)|y|^\beta] \operatorname{sgn} y = 0, \qquad t > 0, \tag{3}$$

where

$$0 < \alpha < 1, \qquad \beta > 1, \tag{4}$$

and a, b: $[0, \infty) \to \boldsymbol{R}$ are continuous.

In Theorem 1 we give conditions which imply that (3) has a solution $\hat{y}$ which is defined on the entire interval $[0, \infty)$ and behaves as $t \to \infty$ like a prescribed solution of the unperturbed equation

$$L_n y = 0, \qquad t > 0. \tag{5}$$

This type of global existence problem for equations of the form $L_n y + f(t, y) = 0$ has recently been studied by the present authors [9]. The theory developed in [9] covers the sublinear case ($b(t) \equiv 0$) as well as the superlinear case ($a(t) \equiv 0$), but not the mixed sublinear-superlinear case ($a(t) \not\equiv 0$ and $b(t) \not\equiv 0$). In this paper a device is presented which enables us to demonstrate the existence of global solutions of the mixed sublinear-superlinear equation (3).

By means of a similar device we find conditions which imply that the mixed sublinear-superlinear elliptic equation

$$\Delta u + \varphi(|x|)u^\lambda + \psi(|x|)u^\mu = 0, \quad x \in \boldsymbol{R}^N, \quad N \geqq 3, \tag{6}$$