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On elliptic equations related to self-similar solutions for nonlinear heat equations

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1. Introduction

This paper studies the existence and nonexistence of global solutions for

$$\Delta w - \left(\frac{1}{2} x \cdot \nabla w + \alpha w\right) + |w|^{p-1} w = 0 \tag{1.1}$$

in $\boldsymbol{R}^n$ for various $p>1$, $\alpha \geqq 0$, where $x \cdot \nabla = \sum_{j=1}^{n} x_j \partial/\partial x_j$.

In [8] we studied the blow-up of solutions of the semilinear heat equation

$$u_t - \Delta u - |u|^{p-1} u = 0. \tag{1.2}$$

We have shown that the asymptotic behavior near the blow-up time is described by special solutions of (1.2) called *backward self-similar solutions*, i.e., functions of the form

$$u(x, t) = (-t)^{-1/(p-1)} w(x/(-t)^{1/2}) \tag{1.3}$$

which solve (1.2) in $\boldsymbol{R}^n \times (-\infty, 0)$; see also [7]. Plugging (1.3) in (1.2) yields an elliptic equation (1.1) for w with $\alpha = 1/(p-1)$.

In [8] we have proved that (1.1) has no bounded global solutions except constant solutions provided $\alpha = 1/(p-1)$ and $n/2 \leqq (p+1)/(p-1)$ (equivalently, $p \leqq (n+2)/(n-2)$ or $n \leqq 2$). In this paper α is considered a *parameter*. It turns out that $1/(p-1)$ is a 'bifurcation point', namely, there is a nonconstant bounded global solution to (1.1) provided $\alpha > 1/(p-1)$ and $n/2 < (p+1)/(p-1)$. For technical reasons we confine ourselves to radial functions, i.e., functions depending only on $r = |x|$. A radial function w is called *radially decreasing* if w is monotonically decreasing as a function of $r > 0$.

THEOREM 1. (*Existence*) *There is a positive radially decreasing solution w of* (1.1) *in $\boldsymbol{R}^n$ provided $\alpha > 1/(p-1)$ and $n/2 < (p+1)/(p-1)$.*

THEOREM 2. (*Asymptotic behavior*) *A positive radially decreasing so-*

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