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Asymptotic behavior of periodic nonexpansive evolution operators in uniformly convex Banach spaces

Dedicated to Professor Isao Miyadera on his sixtieth birthday

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1. Introduction

Let $\{C_t\}_{t\geq 0}$ be a family of nonempty closed convex subsets of a Banach space X and let $U=\{U(t, s): 0\leq s\leq t\}$ be a nonexpansive evolution operator constrained in $\{C_t\}$, i.e. U is a family of mappings $U(t, s): C_s\to C_t$ such that

$$U(t, s)U(s, r) = U(t, r), \quad U(r, r) = I,$$

$$\|U(t, s)x-U(t, s)y\| \leq \|x-y\|$$

for $0\leq r\leq s\leq t$ and $x, y\in C_s$. Such an evolution operator U is said to be T-*periodic* $(T>0)$ if

$$C_{t+T} = C_t \quad \text{and} \quad U(t+T, s+T) = U(t, s) \qquad \text{for} \quad 0 \leq s \leq t.$$

The objective of this paper is to study the asymptotic behavior as $t\to\infty$ of bounded orbits $U(t, 0)x$ defined by a T-periodic nonexpansive evolution operator U. We shall show under appropriate conditions on the space X that if $U(nT+t, 0)x$ is bounded in n for $x\in C_0$ and $t\in[0, T]$, then the sequence $\{U(nT+t, 0)x\}_{n\geq 1}$ is weakly or strongly almost convergent to some T-periodic trajectory $U(t, 0)z$, where z is a point of C_0 with $U(T, 0)z=z$.

In the case of Hilbert spaces this problem was considered for the evolution operator U associated with an initial value problem of the form

$$du(t)/dt + Au(t)\ni f(t), \quad t \geq 0, \quad u(0) = x,$$

by Baillon and Haraux [2], Baillon [1] and Brezis [4], where A is a maximal monotone operator and f is a T-periodic function.

To state our results we recall that X is said to be of type (F) if the norm of X is Fréchet differentiable, namely, for each $x\in X\setminus\{0\}$ the quotient $t^{-1}(\|x+ty\|-\|x\|)$ converges as $t\to 0$ uniformly for $y\in X$ with $\|y\|\leq 1$. It is known that the space L^p is uniformly convex and of type (F) whenever $1<p<\infty$. Further, a sequence $\{x_n\}$ in X is said to be weakly (resp. strongly) almost convergent to x, if $n^{-1}\sum_{i=0}^{n-1} x_{i+k}$ converges weakly (resp. strongly) to x as $n\to\infty$ and the con-