

Stability analysis for free boundary problems in ecology

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Abstract

This paper deals with a free boundary problem for reaction-diffusion equations, which was previously proposed by the authors. The main purpose is to establish the stability theory for stationary solutions for the free boundary problem with fairly general nonlinearity. Our argument is based on the notion of ω -limit set and the comparison principle.

§1. Introduction

In the previous paper [3] the authors have proposed the following free boundary problem for a pair of unknown functions $\{u(x, t), s(t)\}$:

$$(1.1) \quad u_t = d_1 u_{xx} + uf(u), \quad 0 < x < s(t), \quad 0 < t < \infty,$$

$$(1.2) \quad u_t = d_2 u_{xx} + ug(u), \quad s(t) < x < 1, \quad 0 < t < \infty,$$

$$(1.3) \quad u(0, t) = m_1, \quad 0 < t < \infty,$$

$$(1.4) \quad u(1, t) = -m_2, \quad 0 < t < \infty,$$

$$(1.5) \quad u(s(t), t) = 0,$$

$$(1.6) \quad \dot{s}(t) = -\mu_1 u_x(s(t)-0, t) + \mu_2 u_x(s(t)+0, t), \quad 0 < t < \infty,$$

$$(1.7) \quad u(x, 0) = \varphi(x), \quad 0 < x < 1,$$

$$(1.8) \quad s(0) = l,$$

where d_i , m_i and μ_i ($i=1, 2$) are positive constants, f and g are locally Lipschitz continuous functions, φ and l are given initial data, $\dot{s}(t)$ denotes $ds(t)/dt$ and $u_x(s(t)-0, t)$ (resp. $u_x(s(t)+0, t)$) means the limit of $u_x(x, t)$ at $x=s(t)$ from the left (resp. right). Our problem (1.1)–(1.8), which is simply denoted by (P), stems from regional partition phenomena arising in ecology; we consider the situation where two species, that cannot coexist in the same region, are struggling to get their own habitats on the intermediate boundary $x=s(t)$. $u(x, t)$ (resp. $-u(x, t)$) means the population density in $0 < x < s(t)$ (resp. $s(t) < x < 1$). For more details, see [3].