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On wild knots which are weakly tame

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1. Introduction

In this paper, we are concerned mainly with knots, by which we mean topologically embedded circles in the 3-sphere S^3.

Let X be a subset of S^3. Then, X is *PL* if it is a subpolyhedron of S^3, *tame* if $h(X)$ is PL for some homeomorphism $h: S^3 \approx S^3$, and *wild* if it is not tame. Furthermore, X is *locally tame* at $x \in X$ if there are an open set $V \ni x$ in S^3 and a homeomorphism $\phi: V \approx E^3$ such that $\phi(V \cap X)$ is a subpolyhedron of E^3 (E^n denotes the Euclidean n-space), and when X is a knot, X is *locally flat* at $x \in X$ if $\phi(V \cap X) = E^1$ in addition. For a knot $J \subset S^3$, we note that these local properties are equivalent to each other, and consider the closed subset

$$E(J) = \{x \in J \mid J \text{ is not locally tame at } x\} \subset J.$$

Then, Bing's theorem [2] says that J is tame if and only if $E(J)$ is empty.

We shall say that a knot $J \subset S^3$ is *wealky tame* if there is a PL knot $K \subset S^3$ such that the complement $S^3 - K$ is homeomorphic to $S^3 - J$, and *weakly flat* according to Duvall [7] if K is unknotted in addition; and we shall study several properties of such a knot J by taking notice of the set $E(J)$.

The main results are stated as follows.

THEOREM I. *Assume that a knot $J \subset S^3$ is weakly tame, and let U be an open set in J. Then, J is locally tame at every point $x \in U$ if so is at every point $x \in U - C^*$, where C^* is a Cantor set in U.*

COROLLARY. *If a knot $J \subset S^3$ is weakly tame, then $E(J)$ has no isolated points. If J is locally tame at every point $x \in J - C^*$ for a Cantor set $C^* \subset J$ in addition, then it turns out that $E(J)$ is empty and J is tame.*

Theorem I means that $E(J)$ for a weakly tame knot J can not be 0-dimensional. In contrast with this we can find a weakly tame knot J with 1-dimensional $E(J)$: most significant one is given by the following

THEOREM II. *For each PL knot $K \subset S^3$, there is a wild knot $J \subset S^3$ such that $S^3 - J$ is homeomorphic to $S^3 - K$ and J is everywhere wild, i.e., $E(J) = J$.*

A proof of Theorem I using Cannon's characterization of tame arcs in S^3