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Infinite-dimensional algebraic and splittable Lie algebras

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About forty years ago the theory of algebraic Lie algebras of endomorphisms of a finite-dimensional vector space had been developed mainly by C. Chevalley in his works [2, 3, 4, 5] and the theory of splittable Lie algebras of endomorphisms of a finite-dimensional vector space had been developed by the present author in his paper [13]. On the other hand, recently the classical structure theorems of finite-dimensional Lie algebras were extended to a certain kind of locally finite Lie algebras by I. Stewart in his works [1, 11, 12].

In this paper, in connexion with the extended structure theorems we shall generalize the theories of algebraic and splittable Lie algebras to a kind of locally finite Lie algebras of endomorphisms of a not necessarily finite-dimensional vector space.

Let V be a not necessarily finite-dimensional vector space over an algebraically closed field $\mathfrak{k}$ of characteristic 0. For an algebraic endomorphism f of V we consider the Chevalley-Jordan decomposition $f=f_s+f_n$ and the rational decomposition $f_s=\sum \xi_\mu f_{s\mu}$, where $\{\xi_\mu\}$ is a basis of $\mathfrak{k}$ over the prime field. For a Lie algebra L of endomorphisms of V of finite rank we call L splittable (resp. algebraic) if with any element f of L f_s (resp. each $f_{s\mu}$) belongs to L. We shall observe the splittable hull $\hat{L}$ and the algebraic hull $\tilde{L}$ of L and show that $L^2=\hat{L}^2=\tilde{L}^2$ (Theorem 4.6). By making use of a known result on Lie algebras consisting of nilpotent endomorphisms of a finite-dimensional vector space, we shall show that L is splittable (resp. algebraic) if and only if L has a splittable (resp. an algebraic) system of generators (Theorem 6.4). We shall also show that L^2 is always algebraic (Theorem 6.7). Finally we shall generalize several known structure theorems of splittable (resp. algebraic) Lie algebras in [3, 7, 13] to ideally finite splittable (resp. algebraic) Lie algebras of endomorphisms of V (Theorems 7.2, 7.9 and 7.10).

§ 1. Preliminaries

Let L be a not necessarily finite-dimensional Lie algebra over a field $\mathfrak{k}$.

We write $H \leq L$ when H is a subalgebra of L and $H \lhd L$ when H is an ideal of L. We denote by $\zeta(L)$ the center of L.

Let λ be an ordinal. A subalgebra H of L is a λ-step ascendant subalgebra of L, denoted by $H \lhd^\lambda L$, if there exists a series $\{H_\alpha | \alpha \leq \lambda\}$ of subalgebras of L such that