HIROSHIMA MATH. J.
17 (1987), 433–446

On homomorphisms of cocommutative coalgebras and Hopf algebras

Hiroshi YANAGIHARA

(Received January 13, 1987)

Let

$$k \longrightarrow G \xrightarrow{j} H \xrightarrow{\rho} J \longrightarrow k$$

be an exact sequence of cocommutative Hopf algebras over a field k and let C be a cocommutative coalgebra over k. Then it is known that the induced sequence

$$\{e\} \longrightarrow \mathrm{Hom}_{coal}(C, G) \xrightarrow{j_*} \mathrm{Hom}_{coal}(C, H) \xrightarrow{\rho_*} \mathrm{Hom}_{coal}(C, J)$$

of groups is also exact, but that ρ_* is not necessarily surjective. In the paper [2] T. Shudo gave a condition for this homomorphism ρ_* to be always surjective in the case that H is a hyperalgebra. Precisely he showed that ρ_* is surjective for any connected cocommutative coalgebra C over k if and only if the Hopf algebra homomorphism j has a coalgebra retraction $\eta: H \to G$ such that $\eta \circ j$ is the identity map of G.

The main purpose of this paper is to show that the above result for hyperalgebras and connected cocommutative coalgebras is also true for any pointed cocommutative Hopf algebras and coalgebras. In §1 we shall show firstly some properties of cocommutative coalgebras over a field k and coalgebra homomorphisms between them, which are well known in connected cases. Then we shall show in Propositions 4 and 6 that the properties for coalgebra homomorphisms to have coalgebra splittings and coalgebra retractions are colocal in a sense. These results play essential roles in the proof of our main results. In §2 we shall give two theorems. Theorem 1 says that a sequence of pointed cocommutative Hopf algebras over k is exact if and only if the induced sequence of groups consisting of grouplike elements and the sequence of hyperalgebra components of the given sequence are both exact. Theorem 2 is our main result and is a generalization of Shudo's result mentioned in the above.

Throughout this paper we fix a ground field k. All coalgebras, Hopf algebras and their tensor products are defined over k, and our terminology and notations follow those in [3], [4], [5] and [6].