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Global existence of nonoscillatory solutions of perturbed general disconjugate equations

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1. Introduction

Let L_n be the general disconjugate operator

$$L_n = \frac{1}{p_n}\frac{d}{dt}\frac{1}{p_{n-1}}\frac{d}{dt}\cdots\frac{d}{dt}\frac{1}{p_1}\frac{d}{dt}\frac{\cdot}{p_0} \qquad (n \geqq 2), \tag{1}$$

with $p_i>0$ and $p_i \in C[a, \infty)$, $0 \leqq i \leqq n$. Let (1) be in canonical form [10] at ∞; i.e.,

$$\int^{\infty} p_j(t)dt = \infty, \quad 1 \leqq j \leqq n-1. \tag{2}$$

With the operator (1) we associate the quasi-derivatives $L_0u,\ldots, L_{n-1}u$ defined by

$$L_0u = \frac{u}{p_0}; \quad L_ru = \frac{1}{p_r}(L_{r-1}u)', \quad 1 \leqq r \leqq n-1. \tag{3}$$

We give conditions which imply that the equation

$$L_nu + f(t, L_0u,\ldots, L_{n-1}u) = 0 \tag{4}$$

has solutions which behave as $t \to \infty$ like solutions of the unperturbed equation

$$L_nx = 0. \tag{5}$$

Several authors [e.g. 2, 3, 5, 6, 9] have studied perturbed disconjugate equations of the simpler form

$$L_nu + f(t, L_0u) = 0. \tag{6}$$

The more general equation (4), in which the perturbing terms depend also on $L_1u,\ldots, L_{n-1}u$ have been studied in [4], [11] and [12]. However, to the authors' knowledge, all the results previously obtained for nonlinear equations of the forms (4) or (6) are "local" near ∞, in that the desired solutions are shown to exist only for t sufficiently large. Although one of our results given below is a local theorem of this kind which extends a result of Fink and Kusano [4], our main thrust here is in the direction of global results, in which the desired solution is shown to exist on a given interval. This continues a theme —global existence