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Maintenance of oscillations under the effect of a strongly bounded forcing term

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1. Introduction

Consider the forced differential equation

(1) $$L_n x + f(t, x) = h(t)$$

and the corresponding unforced equation

(2) $$L_n x + f(t, x) = 0,$$

where $n \geqq 2$ and L_n is the general disconjugate differential operator defined recursively by $L_0 x(t) = a_0(t)x(t)$ and

$$L_k x(t) = a_k(t)(L_{k-1}x(t))', \quad k = 1, 2,\dots, n.$$

We shall assume without further mention that the functions $a_i(t)$, $i=0, 1,\dots, n$, are positive and continuous on $[t_0, \infty)$ and the operator L_n is in the first canonical form in the sense that

(3) $$\int_{t_0}^{\infty} a_i^{-1}(t)dt = \infty, \quad i = 1, 2,\dots, n-1.$$

In what follows, the set of all real-valued functions $y(t)$ defined on $[t_y, \infty)$ and such that $L_i y(t)$, $i=0, 1,\dots, n$, exist and are continuous on $[t_y, \infty)$ will be denoted by $D(L_n)$.

The purpose of this paper is to examine the oscillatory behaviour of solutions of Eq. (1) by comparing with that of the associated unforced Eq. (2). More precisely, we shall show that the oscillation of solutions of Eq. (1) follows from the oscillation of solutions of Eq. (2) provided that the forcing term $h(t)$ is the n-th "quasi-derivative" of the function $p(t)$ for which $L_0 p(t)$ is strongly bounded in the sense that it assumes its maximum and minimum on every interval of the form $[T, \infty)$, $T \geqq t_0$ (cf. [17]). This means that we can derive oscillation criteria for Eq. (1) from other similar ones which are known for Eq. (2).

Comparison results of this type in the case $a_0(t) = \cdots = a_n(t) = 1$ were first given by Kartsatos [9–12] for the forcings $h(t)$ with the following properties: there exists a continuous function $p(t)$ such that $p^{(n)}(t) = h(t)$ on $[t_0, \infty)$ and either
(I) $\lim_{t\to\infty} p(t) = 0$ and $p(t)$ is oscillatory in $[t_0, \infty)$; or