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Δ-genera and sectional genera of commutative rings

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Introduction

In algebraic geometry and (complex analytic) singularity theory, various "*genera*" are defined for algebraic varieties and singularities to classify them and to study their structure. So it is natural to consider the same problem in commutative ring theory. In [6], we introduced the notion of *genera* and *arithmetic genera* of commutative rings. On the other hand, the classification of (embedded) projective varieties by their *sectional genera* is a quite classical subject in algebraic geometry studied by Enriques, Castelnuovo, Roth and others. This old subject has been recently resurrected and extended to the classification of polarized varieties by their sectional genera (Fujita, Ionescu, Lanteri, Palleschi and others). T. Fujita, among others, introduced the notions of *Δ-genus* and *sectional genus* of a polarized variety, and studied the structure of polarized varieties with low genera.

The aim of this paper is to introduce the notions of *Δ-genera* and *sectional genera* of commutative rings and to study the structure of commutative rings by these genera.

By the way, the non-negativity of the sectional genus and the Δ-genus of a Cohen-Macaulay local ring traces back to Northcott (1960) and Abhyankar (1967). Moreover, the structure of Cohen-Macaulay local rings with low Δ-genera has been studied by J. Sally in detail. Sally's work generalizes the study of rational surface singularities (due to Artin) and minimally elliptic surface singularities (due to Laufer and Wahl).

§1. Δ-genera and sectional genera of polynomial functions

First, we recall some notations and terminologies from [6]. Let $f\colon \boldsymbol{Z}\to\boldsymbol{Z}$ be a polynomial function, i.e., there is a polynomial $P_f\in\boldsymbol{Q}[t]$ such that $f(n)=P_f(n)$ for all $n\gg 0$. We assume, for simplicity, that $f(n)=0$ for all $n<0$. Then there exist (uniquely determined) integers $d\geqq 0$ and e_i $(0\leqq i\leqq d)$, $e_0\neq 0$, such that

$$(\nabla f)(n) := \textstyle\sum_{i=0}^{n} f(i) = e_0\binom{n+d}{d} - e_1\binom{n+d-1}{d-1} + \cdots + (-1)^d e_d,$$

for all $n\gg 0$. Put $d(f)=d$, $e_i(f)=e_i$, $e(f)=e_0$, $g(f)=e_d=(-1)^d P_{\nabla f}(-1)$ and