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Characterization of connection coefficients for hypergeometric systems

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Introduction

In the paper [4] it was shown that a connection problem for the hypergeometric system of linear differential equations

$$(t-B)\frac{dX}{dt} = AX, \tag{0.1}$$

where X is an n-dimensional column vector, $B=\operatorname{diag}[\lambda_1, \lambda_2,\dots, \lambda_n]$ and $A \in M_n(\boldsymbol{C})$, can be solved by the global analysis of the system of linear difference equations

$$(B-\lambda)(z+1)G(z+1) = (z-A)G(z), \tag{0.2}$$

which determines coefficients of power series solutions of (0.1). The method of [4] was effectively applied to solve the connection problem for a system of linear differential equations corresponding to a one-dimensional section of Appell's $F_3(\alpha, \alpha', \beta, \beta', \gamma; x, y)$ in [8]. In this paper, dealing with the complete solution of a connection problem for (0.1) with A which is diagonalizable and has only *two* distinct eigenvalues, we shall clear up the relation between solutions of (0.1) and (0.2), and the structure of connection coefficients in more detail.

In Section 1 we shall be concerned with power series solutions of (0.1) near singularities. In Section 2 we study the system (0.2). In Section 3 we analyze Barnes-integral representations of solutions of (0.1) and characterize the connection coefficients between solutions of (0.1) near a finite singularity and near the infinity. It will be shown that these coefficients are given by solutions of an $(n-1)$-dimensional hypergeometric system obtained from (0.1). In the last section, §4, we deal with some examples.

As for other investigations related to this paper, we refer the reader to [1], [3], [6] and [7].

Hereafter we assume that the diagonal elements λ_j $(j=1,\dots, n)$ of B are all distinct and $A=[a_{jk}]$ is similar to

$$\operatorname{diag}[\overbrace{\mu_1,\dots, \mu_1}^{n1}, \overbrace{\mu_2,\dots, \mu_2}^{n2}] \quad (n_1+n_2=n).$$

Denoting a_{jj} by ν_j $(j=1,\dots, n)$, we furthermore assume the following: