HIROSHIMA MATH. J.
17 (1987), 495–503

Locally inner derivations of ideally finite Lie algebras

Toshiharu IKEDA

(Received January 16, 1987)

Introduction

Let d be a linear endomorphism of a Lie algebra L. We call d a locally inner derivation of L if, for any finite-dimensional subspace F of L, there is an element $x \in L$ such that $yd=[y, x]$ for any $y \in F$. Evidently the set of locally inner derivations of L is an ideal of the derivation algebra $\mathrm{Der}(L)$. It will be denoted by $\mathrm{Lin}(L)$.

C. A. Christodoulou introduced the notion of cofinite Lie algebras by analogy with cofinite groups and investigated their structure in [2]. In group theory locally inner automorphisms and local conjugacy classes of FC-groups have been studied by many authors from various points of view (see for example [3, 4, 6, 9, 10]). In this paper, following their works we study locally inner derivations of ideally finite Lie algebras by making use of the notion of cofinite Lie algebras. In Section 1 we shall show that for a cofinite and ideally finite Lie algebra, its locally inner derivations are precisely those induced by elements of its idealizer in its profinite completion (Theorem 1). In Section 2 we shall show that for an ideally finite Lie algebra L, $\mathrm{Lin}(L)$ is a profinite completion of $\mathrm{Inn}(L)$ for some cofinite topology (Theorem 2), and by using it we shall determine the dimension of $\mathrm{Lin}(L)$ and when $\mathrm{Lin}(L)$ and $\mathrm{Inn}(L)$ coincide over some fields (Theorems 3 and 4, Corollary 2).

1.

We shall be concerned with Lie algebras which are not necessarily finite-dimensional over an arbitrary field $\mathfrak{f}$ of characteristic zero. A Lie algebra L is called a cofinite Lie algebra if it has a topology satisfying the following C1–C4, where $\mathscr{K}(L)$ will denote the set of closed ideals of L of finite codimension, and $\mathscr{T}(L)$ will denote the set of closed vector subspaces of L of finite codimension:

C1. $\cap\{K: K \in \mathscr{K}(L)\}=0$.

C2. For any $H \in \mathscr{T}(L)$, there exists $K \in \mathscr{K}(L)$ such that $K \subset H$.

C3. If H, K are vector subspaces of L such that $H \subset K$ and $H \in \mathscr{T}(L)$, then K is closed.

C4. The set $\{x+U: x \in L, U \in \mathscr{T}(L)\}$ is a subbase of closed sets of L.