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Asymptotic behavior of oscillatory solutions

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1. Introduction

There is an abundance of results in the literature dealing with the oscillation of all solutions of delay differential equations. See, for example, [1]–[6], [8]–[10], and the references cited in [2]. To obtain oscillation results, we usually investigate the asymptotic behavior of the nonoscillatory solutions and then we find conditions on the coefficients and the delays which do not allow such a behavior. By this strategy we are often lead to sufficient conditions for all solutions of certain delay differential equations to oscillate. As a by product, we also learn the asymptotic behavior of the nonoscillatory solutions.

The aim in this paper is to study the asymptotic behavior of the oscillatory solutions of certain delay differential equations of the form

$$(1) \qquad x'(t) + p(t)x(t-\tau) + q(t)x(t-\sigma) = 0, \quad t \geqq t_0$$

and of certain neutral equations of the form

$$(2) \qquad (d/dt)[x(t) - px(t-\tau)] + q(t)x(t-\sigma) = 0, \quad t \geqq t_0.$$

Our results, combined with known oscillation results or with known results about the asymptotic behavior of the nonoscillatory solutions of Eqs. (1) and (2), lead to sufficient conditions for the trivial solution of Eqs. (1) and (2) to be asymptotically stable.

In our opinion, the main contribution of this paper is that it shows how oscillation theory may be used, as another tool, in establishing new stability results for differential equations of diverse nature, like Eqs. (1) and (2) above.

Throughout this paper we will assume that the delays τ and σ in Eqs. (1) and (2) are constants and that the coefficients p and q of Eq. (1) and the coefficient q of Eq. (2) are continuous functions for $t \geqq t_0$ while the coefficient p of Eq. (2) is a constant. With the above assumptions, it follows by the method of steps that, if $\varphi \in C[[t_0 - m, t_0], \boldsymbol{R}]$ is a given initial function where $m = \max\{\tau, \sigma\}$, then Eqs. (1) and (2) have a unique solution x valid for $t \geqq t_0$. By a solution x of Eq. (1) we mean a continuous function for $t \geqq t_0 - m$ such that $x(t) = \varphi(t)$ for $t_0 - m \leqq t \leqq t_0$, $x \in C^1[[t_0, \infty), \boldsymbol{R}]$, and x satisfies Eq. (1) for $t \geqq t_0$. On the other hand, by a solution x of the neutral delay differential equation (2) we mean a continuous function for $t \geqq t_0 - m$ such that $x(t) = \varphi(t)$ for $t_0 - m \leqq t \leqq t_0$, $x(t) - px(t-\tau)$ is