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Strong consistency of the information criterion for model selection in multivariate analysis

Ryuei NISHII[†], Zhidong BAI[††] and Paruchuri R. KRISHNAIAH[††]

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1. Introduction

In the area of model selection, various procedures have been proposed in the literature and their some properties have been examined. In this paper we consider a generalized information criterion (GIC) obtained by the information theoretic approach. According to this procedure, we find the model which minimizes

$$\text{GIC} = -2\log L(\hat{\theta}) + pc_N \tag{1.1}$$

where $L(\hat{\theta})$ is the maximized likelihood and p is the number of parameters. Akaike [1] proposed to take $c_N \equiv 2$, and Rissanen [12] and Schwarz [13] proposed $c_N = \log N$ where N denotes the sample size (see also [2], [8]). Recently Zhao, Krishnaiah and Bai [14] considered the GIC such that

$$\lim_{N\to\infty} c_N/N = 0 \quad \text{and} \quad \lim_{N\to\infty} c_N/\log\log N = +\infty. \tag{1.2}$$

The above criterion is sometimes referred to as the efficient detection (ED) criterion. They used the criterion for the determination of the number of signals under a signal processing model.

In the present paper, we propose to use the ED criterion for certain problems of multivariate analysis. Sometimes the statistician is expected to predict the explanatory variables using some of the response variables under the multivariate regression model. This problem is treated in Section 2 by using the ED criterion, and its consistency is established. Here we may note that Nishii [10] pointed out the inconsistency of Akaike's AIC in calibration. In Section 3 we discuss the selection of variables in discriminant analysis. Our interest is to find the variables which contribute for discrimination between the populations. Section 4 is concerned with canonical correlation analysis, i.e., among two sets of variables we want to find which subsets are important for studying the association between the two sets. The investigations for the above cases will be carried out under a mild condition on the underlying distribution.