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Interior and exterior boundary value problems for the degenerate Monge-Ampere operator

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1. Introduction

This paper deals with interior (exterior) Dirichlet and (Neumann) boundary value problems (b.v.p.) for the real Monge-Ampere (M.A.) equation:

$$\det u_{x_i x_j} = f(|x|)g(|Du|) \qquad \text{in } B_i \text{ (or } B_e), \tag{1}$$

where $B_i = \{x \in \boldsymbol{R}^n; |x| < R\}$, $B_e = \{x \in \boldsymbol{R}^n; |x| > R\}$, $f \geqq 0$, $g(|p|) \geqq 0$.

When we investigate this problem we have in mind the fact that the equation of Gauss curvature of every C^2-smooth surface is given by

$$\det u_{x_i x_j} = K(x)(1 + |Du|^2)^{(n+2)/2} \tag{2}$$

i.e. is of type (1) ($g(t) = (1 + t^2)^{(n+2)/2}$).

Unfortunately the growth of the right-hand side with respect to $|Du|$ leads to the nonexistence results for the Dirichlet b.v.p. even in the case when the Gauss curvature is positive. More precisely, it was shown in [12, 16] that for every $C = \text{const}$ and every $\varepsilon > 0$ there exists C^∞-function φ, $|\varphi| < \varepsilon$ for which the Dirichlet problem for (2) with data $C + \varphi$ on the boundary has no classical convex solution. For this reason only constant boundary data will be considered. This enables us to investigate arbitrary growth of $g(|p|)$. Further on our basic assumption is $g(|p|) \geqq g_0 = \text{const} > 0$ since the more interesting geometric applications satisfy this condition. The degeneration of $g(|p|)$ leads to quite complicated effects like bifurcation of the solutions (see the appendix). We propose complete results for existence, uniqueness and regularity of the classical convex solutions of the M.A. operator with constant data in a ball (B_i, B_e). It is interesting to point out that in this case each classical solution turns out to be a radially symmetric one.

2. Statement of the main results

Because of the lack of space we shall formulate and prove only interior Dirichlet (D_i) and exterior Neumann (N_e) problems for equation (1). By the same methods we can prove similar results for (D_e) and (N_i). Further on the short notations