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# Radial entire solutions of the linear equation $\Delta u + \lambda p(|x|)u = 0$

Dedicated to Professor Tosihusa Kimura on his 60th birthday

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In this paper we are concerned with radial entire solutions of the linear elliptic differential equation

$$(1_\lambda) \qquad \Delta u + \lambda p(|x|)u = 0\,, \qquad x \in \boldsymbol{R}^N\,,$$

where $\Delta$ is the $N$-dimensional Laplacian, $|x|$ denotes the Euclidean length of $x \in \boldsymbol{R}^N$, and $\lambda$ is a positive parameter. We always assume that $N \geqq 3$ and $p$ satisfies

(2) $p \in C[0, \infty)$, $p(t) \geqq 0$ on $[0, \infty)$, and $p(t) \not\equiv 0$ on $[T, \infty)$ for every $T \geqq 0$.

The theorem below requires the further conditions

$$(3) \qquad \int_0^\infty tp(t)\,dt < \infty$$

and

$$(4) \qquad \int_0^\infty t^{N-1}p(t)\,dt < \infty\,.$$

The primary motivation for this paper comes from the observation that very little is known about the asymptotic property of radial entire solutions even for simple linear equations of the form $(1_\lambda)$, whereas there are many results concerning the existence and asymptotic property of positive entire solutions of the *nonlinear* equation

$$(5) \qquad \Delta u + K(x)|u|^{\gamma-1}u = 0\,, \qquad x \in \boldsymbol{R}^N\,, \qquad \gamma \neq 1\,.$$

For some recent literature on equation (5) the reader is referred to the papers [1–3, 5–8] and the references cited therein.

Now let us consider the linear equation $(1_\lambda)$. Assume that (3) is satisfied, and put

$$(6) \qquad P = \int_0^\infty tp(t)\,dt\,.$$