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Generalized random ergodic theorems and Hausdorff-measures of random fractals

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§1. Introduction

Random ergodic theorems were investigated by Kakutani [3], Morita [5] and others. In this paper we establish *generalized* random ergodic theorems which contain the results obtained by Kakutani. A prototype of a generalized random ergodic theorem is found in the proof of the result that the Hausdorff measures of almost all random fractals take a common number.

In Section 2 we show Hausdorff measure's constancy of random fractals to understand a typical generalized random ergodic theorem. Random fractals were investigated by Mauldin-Williams [4], Falconer [1] and Graf [2] and they showed the Hausdorff dimensions of almost all random fractals equal to a constant under some condition. In this section we show that the Hausdorff *measures* of almost all random fractals equal to a constant. In the proof we use a result which is a prototype of a generalized random ergodic theorem.

In Section 3 we develop generalized random ergodic theorems which contain the results obrained by Kakutani [3], and in Section 4 we consider generalized random dynamical systems with random *discrete* parameter, which illustrate the idea developed in Section 3.

§2. Hausdorff measures of random fractals

The starting point for the considerations in the present paper is a scheme used by Graf [2] for producing statistically self-similar fractals. To generate a fractal at random, Graf starts with a probability distribution μ on the set of all N-tuples of contractions of a given bounded separable complete metric space X. First he chooses an N-tuples $(S_1, S_2,\dots, S_N)$ of contractions at random with respect to μ and sets

$$A_1 = \bigcup_{i=1}^{N} S_i(X).$$

For every $i \in \{1,\dots, N\}$, he chooses independently an N-tuples $(S_{i1},\dots, S_{iN})$ at random with respct to μ and sets

$$A_2 = \bigcup_{i=1}^{N} S_i(\bigcup_{k=1}^{N} S_{ik}(X)).$$

He continues this process. Then $K = \bigcup_{n \in \mathbf{N}} \bar{A}_n$ is a random fractal.