HIROSHIMA MATH. J.
19 (1989), 319–327

Error bounds for asymptotic expansions of the maximums of the multivariate t- and F-variables with common denominator

Yasunori FUJIKOSHI

(Received July 20, 1988)

1. Introduction

Let $\boldsymbol{X} = (X_1, \dots, X_p)$ be a scale mixture of a p-dimensional random vector $\boldsymbol{Z} = (Z_1, \dots, Z_p)$ with scale factor $\sigma > 0$, i.e.,

$$(1.1) \qquad \boldsymbol{X} = \sigma \boldsymbol{Z},$$

where $\boldsymbol{Z}$ and σ are independent. Let F_p and Q_p denote the distribution functions of $\boldsymbol{X}$ and $\boldsymbol{Z}$, respectively. Then

$$(1.2) \qquad \begin{aligned} F_p(\boldsymbol{x}) &= P(X_1 \leq x_1, \dots, X_p \leq x_p) \\ &= E_\sigma[Q_p(\sigma^{-1}\boldsymbol{x})], \end{aligned}$$

where $\boldsymbol{x} = (x_1, \dots, x_p)$. The distribution function of $\operatorname{Max}\{X_j\}$ is given by $F_p(x, \dots, x)$. We are concerned with asymptotic expansions of the distribution functions of $\operatorname{Max}\{X_j\}$ and their error bounds in the two important special cases:

(i) $\quad Z_1, \dots, Z_p$ i.i.d. $\sim N(0, 1)$, $\quad \sigma = (\chi_n^2/n)^{1/2}$,

(ii) $\quad Z_1, \dots, Z_p$ i.i.d. $\sim G(\lambda)$, $\quad \sigma = \chi_n^2/n$,

where $G(\lambda)$ denotes the gamma distribution with the probability density function $g(x;\ \lambda) = x^{\lambda-1}e^{-x}/\Gamma(\lambda)$, if $x > 0$, and $= 0$, *if* $x \leq 0$. The random vector $\boldsymbol{X}$ in the case (i) is a multivariate t-variable with common denominator. The random vector $\boldsymbol{X}$ in the case (ii) is essentially equivalent to a multivariate F-variable with common denominator. These distributions are used in simultaneous inferences about the means of normal populations. It may be noted that asymptotic expansions of the distributions of $\operatorname{Max}\{X_j\}$ in the cases (i) and (ii) have been studied by Hartley [6], Nair [7], Dunnett and Sobel [2], Chambers [1], etc. The purpose of this paper is to give a unified derivation of the asymptotic expansions as well as their error bounds.

In Section 2 we give two types of asymptotic approximations for the distribution function of X and their error bounds. The one is newly given, but the other has been given in Fujikoshi and Shimizu [5]. In Section 3 we