HIROSHIMA MATH. J.
19 (1989), 299–307

The orders of the canonical elements in $\tilde{K}(L^n(2^r))$ and an application

Dedicated to Professor Masahiro Sugawara on his 60th birthday

Teiichi KOBAYASHI

(Received July 1, 1988)

§1. Introduction

Let k be a positive integer. For the standard sphere S^{2n+1} in complex $(n+1)$-space C^{n+1}, let T_k: $S^{2n+1} \to S^{2n+1}$ be the fixed point free transformation of period k defined by

$$T_k(z_0, z_1, \cdots, z_n) = (\lambda(k)z_0, \lambda(k)z_1, \cdots, \lambda(k)z_n),$$

where $\lambda(k) = e^{2\pi i/k}$, $\sum_{j=0}^{n}|z_j|^2 = 1$, $z_j \in C$ $(0 \leq j \leq n)$. Then T_k generates the cyclic group Z_k of order k. The orbit space S^{2n+1}/Z_k is the standard lens space mod k, and is denoted by $L^n(k)$.

Let η be the canonical complex line bundle over $L^n(2^r)$, and define the canonical elements in the reduced K-ring $\tilde{K}(L^n(2^r))$ of $L^n(2^r)$ by

$$\sigma(s) = \eta^{2^s} - 1 \ (0 \leq s \leq r), \ \sigma = \sigma(0).$$

The purpose of this paper is to determine the order of $\sigma(s)^i \in \tilde{K}(L^n(2^r))$.

THEOREM 1.1. *Let s and i be integers such that $0 \leq s \leq r$ and $1 \leq i \leq [(n-1)/2^s] + 1$. Then the element $\sigma(s)^i$ in $\tilde{K}(L^n(2^r))$ is of order $2^{r-s+1+[(n-1)/2^s]-i}$.* (Here $[x]$ is the integer part of a real number x.)

For $s = 0$, the result is obtained by T. Kawaguchi and M. Sugawara [2, Theorem 1.1(i)].

Let k be a positive integer. Then a fixed point free transformation $\bar{T}_2$: $L^n(k) \to L^n(k)$ of period 2 is induced by T_{2k}: $S^{2n+1} \to S^{2n+1}$, since $(T_{2k})^2 = T_k$.

As an application of Theorem 1.1, we prove

THEOREM 1.2. *Let q and t be odd integers. If there is a Z_2-equivariant map from $(L^n(2^r q), \bar{T}_2)$ to $(L^m(2^s t), \bar{T}_2)$, then $[(n-1)/2^r] \leq [(m-1)/2^s]$.*

In case $s = 0$ and $t = 1$, this result follows also from the mod 2 part of a theorem due to H. J. Munkholm and M. Nakaoka [9, Theorem 4] and K. Shibata [10, Theorem 7.4] (cf. J. W. Vick [11, Corollary (3.3)]), since $[(n-1)/2^r] \leq m-1$ if and only if $n \leq m \cdot 2^r$.