HIROSHIMA MATH. J.
19 (1989), 251–258

Moduli space of 1-instantons on a quaternionic projective space HP^n

Hideo DOI and Takayuki OKAI

(Received May 20, 1988)

Introduction

The moduli space of 1-instantons on $S^4 = HP^1$ is isomorphic to $Sp(2)\backslash SL(2, H)$ ([2], [3], [9]). The main purpose of this paper is to generalize this basic fact to the case of HP^n. More precisely, we consider self-dual connections, i.e. solutions to a first order equation which is a reduction of the Yang-Mills equation given by physicists [4], [20].

At present a general theory for self-dual connections on quaternionic Kähler manifolds is developed by M. Mamone Capria & S. M. Salamon [12] and T. Nitta [15]. Thus it would be worthwhile to study self-dual connections concretely. In this point of view E. Corrigan, P. Goddard & A. Kent [5] have provided an interesting family of self-dual connections on HP^n, as a generalization of the ADHM construction. They have also counted the number of parameters of this family. For 1-instantons (see § 1), from the table of H. T. Laquer [11], we know that this number coincides with the nullity of the second variation of the Yang-Mills functional at the canonical connection for the symmetric space $Sp(n+1)/Sp(1) \times Sp(n)$. However, even in this case, the completeness of the ADHM construction is a problem [5]. In Theorem 1.1, we will give an affirmative answer to this, using a result in algebraic geometry due to H. Spindler [19]. In Theorem 1.2, we will give a compactification of the moduli space of 1-instantons. In Theorem 1.3, we will examine the convergence of the Yang-Mills action densities.

1. Notation and the results

We begin with a review of quaternionic geometry (for details, see [12], [14, 15], [16, 17, 18]). Let M^{4n} be a *quaternionic Kähler manifold*. By definition its holonomy group is contained in $Sp(n)\cdot Sp(1) \subset SO(4n)$. Note that the natural representation of $GL(n, H) \times Sp(1)$ on $\Lambda^2(C^{2n} \otimes C^2)$ is decomposed to $\Lambda^2 C^{2n} \otimes S^2 C^2 + S^2 C^{2n} \otimes \Lambda^2 C^2$. Accordingly, we have a decomposition $\Lambda^2 T^*M \otimes C = \Lambda_2 + \Lambda_0$. Let E be a complex unitary vector bundle over M with a unitary connection D. We assume that its curvature form $F(D)$ is a section of $\Lambda_0 \otimes \mathrm{u}(E)$. Then D is said to be *self-dual*. Note that D becomes a Yang-Mills connection. If a transformation $g: M \to M$ preserves the $GL(n, H)\cdot$